Name _____ Date _____ Class # _____ Block _____

Intro to Calculus

Workshop 12 (After Homework 15)

1. Simplify, leaving no negative exponents in your final answer. Combine like terms.

   a. $5x\left(x^2-3\right)(x+2)^{\frac{-5}{8}}+4(x+2)^{\frac{19}{8}}\left(-\frac{1}{2}\right)$

   b. $\left(2x^2-3x+1\right)(4)(3x+2)^3(3)+(3x+2)^4(4x-3)$

   c. $\left(x^2-4\right)^{\frac{1}{2}}(3)(2x+1)^2(2)+(2x+1)^3\left(\frac{1}{2}\right)\left(x^2-4\right)^{\frac{-1}{2}}(2x)$

   d. $(3x+1)^6\left(\frac{1}{2}\right)(2x-5)^{\frac{-1}{2}}(2)+(2x-5)^{\frac{1}{2}}(6)(3x+1)^5(3)$

   e. $\dfrac{\sqrt{x\sqrt{\frac{x}{y}}}}{\sqrt[3]{\frac{y}{x^2}}}$

2. The following are represent some of the incorrect responses to the question "Explain the meaning of the number *e*" from Test 5. Explain what is wrong with each of them and then write your best answer to the original question.

   a. *e* is the greatest number a continuous growth will reach.

   b. *e* is an asymptotic point.

   c. *e* is the value of $\left(1+\frac{1}{n}\right)^n$.

   d. *e* is the amount of money you get if you started with $1 and compounded the interest continuously.

3. Suppose that $10 was invested in an account whose return was 16% per year.

   a. What would be the total in the account after 1 years if the interest was compounded every 3 months? Make a growth diagram (the branching boxes thing like we did before) to show how this works. Then use an equation to calculate it.

   b. What would be the total in the account after 3 years if the interest was compounded every day?

   c. What would be the total in the account after 3 years if the interest was compounded continuously?

4. Simplify.

   a. $4^{\log_{\frac{1}{\sqrt{2}}}\frac{1}{3}}$ b. $\left(\frac{\sqrt{3}}{9}\right)^{\log_{27}(64)}$ c. $\left(\sqrt[3]{e}\right)^{\ln\left(\frac{8}{125}\right)}$

5. Solve for x. Use your calculator to find an approximate solution AFTER isolating x.

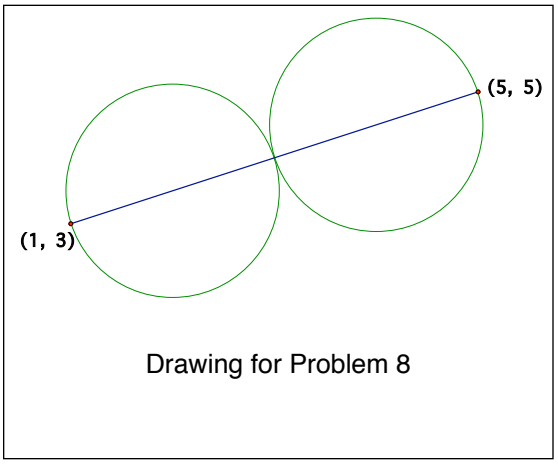
   a. $10^{2x+1} = 4^{x-1}$ b. $3^{2x+1} = 6^{x-3}$ c. $3(1.06)^x = 25$ d. $2^{x^2-x} = 4$

6. At the start of an experiment the mass of a certain subject was 50 grams. Three days later the mass was 10 grams. Assuming that the loss of mass can be modeled by the equation $m(t) = Cb^t$, at what time was the mass 25 grams? (In other words, find the half-life.)

7. A substance loses one quarter of its original mass in 8 days. Find its half-life (that is the time it takes for the mass to become half of its initial amount) assuming it can be modeled by $f(t) = Cb^t$. Round appropriately.

8. Two circles have the same radius and line up as shown. Find the equations of the circles.

9. Find an equation for the smallest circle that is centered at the origin and contains the circle $x^2 - 6x + y^2 - 8y = -21$ (i.e., no point on the circle ($x^2 - 6x + y^2 - 8y = -21$ lies outside of the circle that you are looking for). Justify!



Drawing for Problem 8

10. Given that $\log_2(3) = a$, evaluate $\log_2(81)$.

11. If $\log_a(b) = 4$, evaluate $\log_a\left(\frac{a}{b}\right)$.

12. If $ab = 5$, evaluate $\log_5\left(a^2b^2\right)$.

13. If $a = \frac{b^3}{c^2}$, expand $\log(a)$.

14. Given $\log(x) = 2$, evaluate $\log\left(100x^4\right)$.

15. If $\log_{36}(x) = A$, then $\log_6(x) = ?$

16. If $\log(a) = 0.6$, and $\log(b) = 0.4$, evaluate $\log\left(\frac{a^2}{b}\right)$.

17. If $\log_4(x) = 15$, then $\log_4\left(\frac{4}{x^{-2}}\right) = ?$

18. If $\log(x) = 4.3$, and $\log(y) = -0.8$, then $\frac{y}{x} = ?$

19. If $\log_a(32) = 5$, and $\log_4(b) = 2.5$, then $\log_4(ab) = ?$

20. If $\log_a(x) = b$, then $\log_a\left(\frac{a}{x^3}\right) = ?$

21. If $5^x = b$, evaluate $25^x \cdot 125^{3x}$.

22. If $3^x = a$, evaluate $\log_3\left(a^4\right)$.

23. Given $\frac{y^3}{xz^3} = 3$, evaluate $\log_3\left(\frac{1}{x}\right) + 2\log_3(y) - \log_3\left(z^3\right)$.

Selected Hints and Solutions

1. a. $\dfrac{3x^3-12x^2-39x-16}{(x+2)^{5/8}}$

   b. $(3x+2)^2(36x^2-37x+6)$

   c. $\dfrac{(2x+1)^2(8x^2+x-24)}{(x^2-4)^{1/2}}$

3. a. The best way to make these sorts of diagrams is to think about how many lines you will need at the very end, set these up, and then work backwards from there. In this problem you have to compound 4 times (12 months ÷ 3 months), so you will need 16 lines at the very end. The next thing you have to figure out is the amount of growth per compounding. Since there are 4 compounds and the annual growth is 16%, each compounding will have a growth of 4%, or 0.04. In the end you should have a total of $11.70. A good website to help you is http://www.webmath.com/compinterest.html

   c. Remember that since you are now compounding continuously you will need to use e. You should end up with approximately $16.17.

4. a. We want to find a numerical value for $4^{\log_{\frac{1}{\sqrt{2}}}\frac{1}{3}}$.

   Let $\log_{\frac{1}{\sqrt{2}}}\frac{1}{3}=Q$, then

   $$2^{-\frac{Q}{2}}=\frac{1}{3}$$
   $$2^{Q}=3^2$$

   Now we want to know the value of 4^Q. Hence

   $$4^Q=2^{2Q}$$
   $$=\left(3^2\right)^2$$
   $$=81$$

Therefore, $4^{\log_{\frac{1}{\sqrt{2}}}\frac{1}{3}} = 81$.

4. b. Start by letting $\left(\frac{\sqrt{3}}{9}\right)^{\log_{27}(64)} = \left(\frac{\sqrt{3}}{9}\right)^{x}$.

Your final answer should be $\left(\frac{\sqrt{3}}{9}\right)^{\log_{27}(64)} = \frac{1}{8}$.

5. b. $x \doteq -15.97$

5. d. $x = 2, -1$

10. $4a$	15. 2A	20. $1-3b$
11. -3	16. 0.8	21. b^{11}
12. 2	17. 31	22. $4x$
13. $3\log(b)-2\log(c)$	18. $10^{-5.1}$	23. 1
14. 10	19. 3	