Good luck to: _____ Block: A2 A3 A4 B2 B3 B4 Date: _____ ID: A

Intro to Calculus Test 6 Mr. Holcomb 2008/2009
Ice Cream or Cake?

Problem

1. (6 points) If the angle α in standard position terminates in Quadrant II, in what quadrants do the following angles terminate?

   a. $\alpha - \pi$ b. $-\alpha$ c. $\alpha + \frac{3\pi}{2}$

   d. $\pi - \alpha$ e. $\frac{\pi}{2} - \alpha$ f. $-\alpha - \frac{15\pi}{2}$

   .

2. (15 points) Find all values of the variable in the interval $\left[0, 2\pi\right)$ which will make the following true. Justify with clear and complete work.

   a. $6\cos(2\alpha) + 3 = 0$ b. $4\sin^2(2\alpha) = 3$

   c. $2\sin(\beta)\cos(\beta) = \sin(\beta)$

3. (10 points) Without using your calculator, evaluate the following over the interval $\left[0,2\pi\right)$. Use a unit circle drawing **for each** to help justify.

   a. $\cos^{-1}\left(\cos\left(\frac{-8\pi}{5}\right)\right)$

   b. $\sin^{-1}\left(\sin\left(\frac{22\pi}{7}\right)\right)$

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4. (6 points) Without using your calculator, find all the numbers t for which $\sec(t) = -2$. Justify.

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5. (6 points) Without using your calculator, find all of the angles θ for which $-2\pi \le \theta \le 2\pi$ and $\cos(\theta) = \cos\left(\frac{30\pi}{7}\right)$. Justify.

6. (6 points) Simplify: $\frac{1}{1-\sin(\alpha)} + \frac{1}{1+\sin(\alpha)}$

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7. (10 points) Show that the following identity is always true:

$$\frac{\csc^3(\alpha) - \csc(\alpha) + \cot(\alpha)}{\csc(\alpha)} = \cot^2(\alpha) + \cos(\alpha)$$

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8. (16 points) Let $\sin(\theta) = -0.8$ and $-\pi \le \theta \le \frac{-\pi}{2}$. Find each of the following without using a calculator.

a. $\cos(\theta)$

b. $\cot(\theta)$

c. $\cos(-\theta)$

d. $\sin(\theta + \pi)$

e. $\cos\left(\theta + \frac{\pi}{2}\right)$

f. $\sin\left(\theta - \frac{\pi}{2}\right)$

g. an angle β (expressed in terms of θ) for which $\cos(\beta) = 0.8$ and $\sin(\beta) > 0$

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9. (5 points) Without using your calculator, find all angles $t, 0 \le t \le 2\pi$, for which $\cos(t) = \sin\left(\frac{3\pi}{7}\right)$.

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10. (5 points) A CD has a radius of 6 cm. As the CD spins in a CD player, a point on the edge of the CD is moving 188.5 meters per second. How fast is the CD spinning, in radians per second? (Approximate to the nearest 0.1 radian per second.)

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11. (10 points) Suppose that $\sin(\alpha) = \frac{-7}{25}$. Without using your calculator find all possible values of the following. Justify.

a. $\sin\left(\alpha + \frac{\pi}{2}\right)$

b. $\sec\left(\alpha + \frac{\pi}{2}\right)$

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12. (5 points) Using your calculator to help, find all solutions such that $0 \le \theta < 2\pi$ for $\sin(\theta) = -0.2$.

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13. (5 points) Suppose that $\cos(\alpha) = \frac{-8}{17}$ and $\sin(\alpha) = \frac{15}{17}$. Find all angles, β, in terms of α, for which $\cos(\beta) = \frac{-15}{17}$ and $\sin(\beta) = \frac{-8}{17}$. Justify.

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14. (15 points) Solve for x. Leave answers in exact form, simplified.

a. $\log(x-2) + \log(x-3) - \log(2) = \log(x)$

b. $5^{2x+1} = 7$

c. $\log_4\left(\log_5(x)\right) = \ln\sqrt{e}$

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Answer Section

PROBLEM

1. ANS:
   a. Q4 b. Q3 c. Q1

   d. Q1 e. Q4 d. Q4

   PTS: 6
2. ANS:
   a. π/3, 2π/3, 4π/3, 5π/3-- note: the domain of α [0,2π) so the domain of 2α is [0, 4π)

   b. π/6, π/3, 2π/3, 5π/6, 7π/6. 4π/3, 5π/6, 11π/6-- note: the domain of α [0,2π) so the domain of 2α is [0, 4π)

   c. π/3, 5π/3, 0, π-- note: if both sides of the equation are just divided by $\sin(\beta)$, then a pair of solutions is missed. You must factor-- just like back in agl. 1.

   PTS: 15
3. ANS:
   a. 2π/5, 8π/5 b. 8π/7, 13π/7

   PTS: 10
4. ANS:
   $\frac{2\pi}{3} + 2\pi n \,\middle|\, n \in Z$ and $\frac{4\pi}{3} + 2\pi n \,\middle|\, n \in Z$

   PTS: 6
5. ANS:
   $\frac{-12\pi}{7}, \frac{-2\pi}{7}, \frac{2\pi}{7}, \frac{12\pi}{7}$

   PTS: 6

6. ANS:

$$\frac{1}{1-\sin(\alpha)}+\frac{1}{1+\sin(\alpha)}=\frac{1}{y-1}+\frac{1}{y+1}$$

$$=\frac{1+y+1-y}{1-y^2}$$

$$=\frac{2}{x^2}\,(\text{by Pyth. ID})$$

$$=\frac{2}{\cos^2(\alpha)}$$

PTS: 6

7. ANS:

$$\frac{\csc^3(\alpha)-\csc(\alpha)+\cot(\alpha)}{\csc(\alpha)}=\cot^2(\alpha)+\cos(\alpha)$$

$$\left[\frac{1}{y^3}-\frac{1}{y}+\frac{x}{y}\right]\div\frac{1}{y}$$

$$\frac{1}{y^2}-1+x$$

$$\frac{1}{y^2}-\frac{y^2}{y^2}+\frac{xy^2}{y^2}$$

$$\frac{1-y^2}{y^2}+\frac{xy^2}{y^2}$$

$$\frac{x^2}{y^2}+x(\text{by Pyth. ID})$$

PTS: 10

8. ANS:

a. -0.6 b. 0.75

c. -0.6 d. 0.8

e. 0.8 f. 0.6

PTS: 16

9. ANS:
$\frac{\pi}{14}, \frac{27\pi}{14}$

PTS: 5

10. ANS:
3141.7 radians per second

PTS: 5

11. ANS:
a. $\frac{-24}{25}, \frac{24}{25}$ b. $\frac{25}{7}$

PTS: 10

12. ANS:
$\theta \approx \pi + .2, 2\pi - .2$

PTS: 5

13. ANS:
$\beta = \alpha + \frac{\pi}{2}$

PTS: 5

14. ANS:
a. x= 6 (Note: x=1 will be out of the domian)

b. x = 25

c. $x = \frac{1}{2}\left(\frac{\ln(7)}{\ln(5)} - 1\right)$

PTS: 15