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Intro to Calculus

Workshop 3[1]

1. a. Find the equation of the circle of radius 5 that intersects the *x*-axis only once and intersects the *y*-axis only once.

   b. How many circles are there that could be an answer to part (a) of this problem?

2. The shape of an indoor track is determined by starting with a rectangle and placing a pair of semicircles on opposite sides of the rectangle. The track then runs around the resulting shape. If the original rectangle is a square, find a rule that describes the length of the track as a function of the length of the side of the square. Then use your rule to show that the side of the square must have been about 239 meters if the total length of the track was 1500 meters.

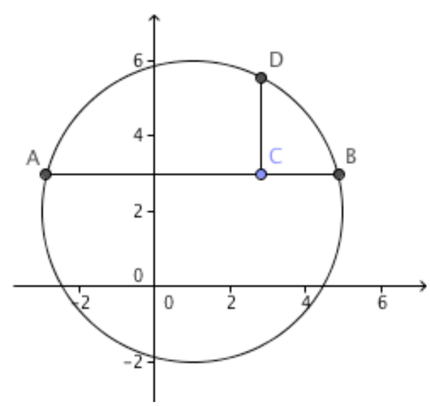
3. Find the domain of the following functions. Express the domain using interval notation. Justify with clear and complete work. Use your calculator to check ***after*** you have found the domain algebraically.

   a. $f(x)=\sqrt{3-|x|}$ b. $f(x)=\sqrt{1-|x-2|}$ c. $f(x)=\sqrt{2-\sqrt{(x-4)^2}}$

4. Consider the diagram shown where the circle has a radius of 4 units and is centered at the point (1, 2). Further, point *C* is on $\overline{AB}$ (which is 3 units above the *x*-axis) such that point *C* is free to slide back and forth on $\overline{AB}$. Lastly, $\overline{CD}$ remains perpendicular to $\overline{AB}$ throughout the movement of point *C*.

   a. What is the greatest possible length of $\overline{CD}$? At what value of *x* does this occur? Justify.

   b. Suppose that an alarm bell is suppose to ring whenever point *C* gets closer than 0.5 units of either endpoint of the segment. What are the locations (approximate to within 0.01 units) of point *D* for which the alarm rings? Justify with clear and complete work.



Drawing for Problem 4

Sketch at http://holcombmath.com/sketches/I2C_geogebra_sketches/I2C_WS_3_Problem_4.html

[1] This work extensively borrows from work of Prof. Scott Farrand, CSU Sacramento

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c. Create a rule that allows you to predict *CD* when you know the location of point *C*. Justify with clear and complete work.

d. Use the rule you created in part (c) of this problem to confirm your result from part (a) of this problem.

e. Create a rule that allows you to predict the location of point *C* when you know the length of point *D*. Justify with clear and complete work.

f. Use your rule from part (e) of this problem to show that $CD = 1$ when $x \approx -2.46$ or $x \approx 4.46$.

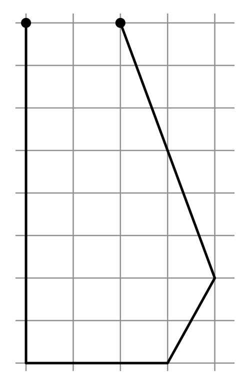
g. Considering the situation you are dealing with in this problem, are either of the rules you wrote for parts (c) and (e) of this problem functions? Explain.

5. Determine whether *y* is a function of x by isolating *y*. If it is a function, state the domain using interval notation. Justify with clear and complete work.

a. $y - xy = x^2$ b. $y + \frac{x}{y} = 3$ c. $\sqrt{y} = x^2 + 1$

6. Create two functions, *f* and *g*, such that $f\left(g(x)\right) = \frac{1}{\sqrt{\left(|x-4|\right)^2 - 1}}$.

7. Imagine we could have flat bottles. One such bottle is shown. Further imagine that water is being poured into the "top" of the bottle.



"Flat bottle" for problem 7

a. Explain why the area that is filled is equal to 7 square units when the height of the water is 2 units.

b. Explain why the area that is filled is equal to 25 square units when the height of the water is 8 units.

c. Create a table showing the relationship between the height of the water and the area that is filled for heights 1 through 8. Use fractions and your values should be exact, not estimates.

d. Create a graph to represent the area filled as a function of the height of the water in the bottle.

e. Should the graph from part (e) be linear? How does the shape of the bottle play a role?