

Intro to Calculus

Workshop 27 [After "Who's the Boss?"]

1. In the lesson for homework 27 you were asked to determine the order in which the functions below finished in a "race to infinity". The table shows the place of some of the functions during this race. As you saw the order changes from $x = 1$, to $x = 10$, to $x = 100$, and then finally when $x \rightarrow +\infty$.

Function
$a(x) = 50\sqrt{x}$
$b(x) = x^{10}$
$c(x) = 100x^2$
$d(x) = 800\log(x)$
$e(x) = 2^x - 1000$
$f(x) = x$
$g(x) = 1.1^x$
$h(x) = 20x^3$

Place	$x = 1$	$x = 10$	$x = 100$	$x \rightarrow +\infty$
First	$c(x)$		$e(x)$	$e(x)$
Second	$a(x)$	$h(x)$		
Third	$h(x)$	$c(x)$	$h(x)$	
Fourth			$c(x)$	$h(x)$
Fifth		$a(x)$		$c(x)$
Sixth		$e(x)$		
Seventh	$e(x)$		$a(x)$	$a(x)$
Eighth	N/A			

- Complete the table with the other functions.
- What was the approximate x value at which $a(x)$ was tied with $e(x)$? (Use a graph-- you can't solve this algebraically!)
- Which function(s) passed function h between $x = 10$ and $x = 100$. At approximately what value of x were these two functions tied?
- What was the x -value after which the order of the functions did not change?

Intro to Calculus: Workshop 27

2. Let $f(x) = \frac{(x^2 - 1)}{(x + 1)}$.
- Before you pick up your calculator, predict what the graph of f will look like. Then verify with your calculator. Sketch the graph.
 - Evaluate $f(-1)$
 - Evaluate $f(1)$
 - Evaluate $\lim_{x \rightarrow -1} f(x)$
 - Evaluate $\lim_{x \rightarrow +1} f(x)$
3. a. For each of the following rational functions: state the domain, sketch the graph and find $\lim_{x \rightarrow 2} f(x)$.
- $f(x) = \frac{2x - 4}{x - 2}$
 - $f(x) = \frac{x^2 + x - 6}{x - 2}$
 - $f(x) = \frac{x - 2}{x^2 - 4}$
 - $f(x) = \frac{x - 2}{\sqrt{x - 2}}$
- b. Find a rational function g whose domain is $x \in (-\infty, 1) \cup (1, 2) \cup (2, \infty)$ and such that $g(x) = x^2$ wherever $g(x)$ is defined.
4. Compute the following limits analytically. Verify by graphing where possible.
- $\lim_{x \rightarrow -1} \frac{|x|}{x}$
 - $\lim_{x \rightarrow 1} \frac{x^3 - x^2 + x - 1}{x^2 - 1}$
 - $\lim_{x \rightarrow 0} \sqrt{x}$
 - $\lim_{x \rightarrow 0} \frac{(1+x)^2 - 1}{(1+x)^3 - 1}$
 - $\lim_{x \rightarrow 2} \frac{\sqrt{x} - \sqrt{2}}{x - 2}$
 - $\lim_{x \rightarrow 2} f(x)$ where $f(x) = \begin{cases} 5 + x, & x < 3 \\ 4, & x \geq 3 \end{cases}$
 - $\lim_{x \rightarrow 2} f(x)$ where $f(x) = \begin{cases} x^2, & x \geq 2 \\ \frac{x^2 - 4}{x - 2}, & x < 2 \end{cases}$
 - $\lim_{h \rightarrow 0} f(2 + h)$ where f is a function for which $\lim_{x \rightarrow 2} f(x) = 4$.

Intro to Calculus: Workshop 27

5. For each of the four cases below, sketch a graph of some function that satisfies the stated condition.

a. $\lim_{x \rightarrow 2} f(x) = 3$ and $f(2) = 4$

b. $\lim_{x \rightarrow 0} f(x)$ does not exist and $|f(x)| < 2$ for all x .

c. $\lim_{x \rightarrow 0} f(x) = f(0) + 1$

d. $\lim_{x \rightarrow +\infty} f(x) = 1$ and $\lim_{x \rightarrow -\infty} f(x) = -1$

6. Determine the following limits analytically. Check by graphing.¹

a. $\lim_{x \rightarrow +\infty} f(x)$ where $f(x) = \frac{\sqrt{x} + 3}{x - 4}$

b. $\lim_{x \rightarrow -\infty} f(x)$ where $f(x) = \frac{x^2 + 6x + 8}{x^2 - 5x - 14}$

c. $\lim_{x \rightarrow +\infty} f(x)$ where $f(x) = x^2 \left[\sqrt{2 + \left(\frac{1}{x^2}\right)} - \sqrt{2} \right]$ (It's not 0.)

d. $\lim_{x \rightarrow +\infty} f(x)$ where $f(x) = \frac{\sqrt{\pi x^2 - 1}}{x + 3}$ (Hint: Multiply by conjugate of numerator, then divide through by the highest power of x .)

e. $\lim_{x \rightarrow +\infty} f(x)$ where $f(x) = x - \sqrt{x^2 - x}$

¹ Farrand/Poxon p.59

Intro to Calculus: Workshop 27

Solutions

6. a) 0, b) 1, c) $\frac{\sqrt{2}}{4}$, d) $\sqrt{\pi}$, e) $\frac{1}{2}$