

Intro to Calculus

Workshop 25 [After Lesson 2 for Polar]

In the next few problems you will be shifting polar graphs and figuring out how to rotate them about the pole.

1. It is nice to have the following two equations shown on separate graphing calculators so that you can look at them side by side: $r = 2 \sin(\theta)$, $r = 2 \sin\left(\theta - \frac{\pi}{4}\right)$.
2. Make a prediction about what the following graphs of the following polar equations will look like and how they will be related to each other. Then graph them on your calculator to confirm your predictions: $r = 3 \sin(\theta)$, $r = 3 \sin\left(\theta + \frac{\pi}{4}\right)$.
3. By thinking about the previous problem, write the equation of the new graph formed when $r = 1 + 3 \sin(\theta - \pi)$ is rotated by an angle of π .
4. Predict what the graph of $r = \frac{\sin(\theta)}{\cos^2(\theta)}$ will look like. Sketch your prediction. Then verify with your graphing calculator.
5. Translate the polar equation in the previous problem into rectangular coordinates. What kind of conic section is being represented?
6. Predict what the graph of $r = \frac{\sin(\theta - \pi)}{\cos^2(\theta - \pi)}$ will look like. Sketch your prediction. Then verify with your graphing calculator.
7. Create a polar equation which will generate a graph of the parabola $y = x^2$ rotated $\frac{\pi}{4}$ radians about the origin. Use your calculator to help. Sketch the graph.
8. Create a polar equation which will generate a graph of the parabola $y = (x - 1)^2 + 2$ rotated $\frac{\pi}{2}$ radians about the origin. Use your calculator to help. Sketch the graph.
9. Create a polar equation which will generate a circle which whose center is located at $(1, -1)$ and whose radius is 3.

Intro to Calculus: Workshop 25

10. Suppose that you are given the graph of $r = f(\theta)$ where f is some function. Write an equation in terms of f which would:

- Rotate the graph of f by $\frac{\pi}{2}$ radians.
- Rotate the graph of f by $-\frac{\pi}{2}$ radians?

11. Transform the following equations from rectangular to polar form.

- $6x - x^2 = y^2$
- $2x + 3y = 5$
- $x^2 + y^2 = 7$
- $y = x^2$

12. Transform the following equations from polar to rectangular form.

- $r = \sin(\theta)$
- $r + 3\sin(\theta) = 0$
- $r(2\cos(\theta) + \sin(\theta)) = 4$

13. Gold Star Problem: Using algebra, clearly and completely show that the polar equation

$$r = \frac{\sin\left(\theta + \frac{\pi}{4}\right)}{\cos^2\left(\theta + \frac{\pi}{4}\right)} \text{ is equivalent to } \frac{(x-y)^2}{x+y} = \sqrt{2}.$$

14. A boat traveling due north sees another boat traveling due east cross 800 meters directly in front of it at time $t = 0$ seconds. The first boat is traveling at a rate of 4 meters per second and the other boat is traveling at a rate of 3 meters per second.

- Find the degree measure of the angle (from due north with clockwise being the positive direction) that a person on the first boat needs to angle his telescope to so that it is pointed directly at the second boat when $t = 3$ seconds. Round your answer to the nearest 0.1 unit.
- Find the degree measure of the angle (from due north with clockwise being the positive direction) that a person on the first boat needs to angle his telescope to watch the second boat. Express your answer in terms of t , the number of seconds since the second boat was due north of the first.
- If the boat that is traveling north wants to catch the boat that is traveling east, what heading (from due north with clockwise being the positive direction) will the boat need to take and how long will it take to catch up to the eastbound boat?