

# Intro to Calculus

## Workshop 19<sup>1</sup>

1. Match the following expressions with all of the letters on the diagram to which they correspond.

i.  $\cos(\alpha) = \frac{1}{2}$

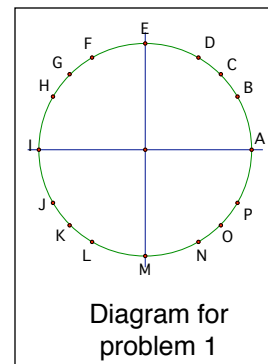
ii.  $\cos(\alpha) = -\frac{\sqrt{3}}{2}, \pi < \alpha \leq 2\pi$

iii.  $\sin(\alpha) = \cos\left(\alpha + \frac{\pi}{2}\right), 0 < \alpha \leq \pi$

iv.  $\sin(\alpha) = -\frac{\sqrt{3}}{2}$

v.  $\sin\left(\alpha + \frac{\pi}{2}\right) = \cos\left(\alpha - \frac{\pi}{2}\right), 0 \leq \alpha < \pi$

vi.  $\cos(-\alpha) = \sin(\alpha + \pi)$



2. Let  $P$  represent the point of intersection of the terminal side of the central angle  $\alpha$  (whose initial side is the positive  $x$ -axis) and the unit circle. Find **all** values for  $\alpha$  which make each of the following true. Justify with a unit circle drawing.

a.  $P\left(\frac{-1}{2}, \text{---}\right)$  in  $Q_{III}$ ,  $0 \leq \alpha$

b.  $P\left(\frac{\sqrt{3}}{2}, \text{---}\right)$  in  $Q_{IV}$ ,  $\alpha < 0$

c.  $P\left(\frac{-\sqrt{3}}{2}, \text{---}\right)$  in  $Q_{III}$ ,  $\alpha > 0$

d.  $P\left(-\frac{\sqrt{2}}{2}, \text{---}\right)$  in  $Q_{II}$ ,  $\alpha < 0$

3. Simplify, without using your calculator. Justify with a unit circle drawing, Find all solutions.

a.  $\cos^{-1}\left(\cos\left(\frac{16\pi}{11}\right)\right)$

b.  $\sin^{-1}\left(\sin\left(\frac{13\pi}{5}\right)\right)$

<sup>1</sup> Based on work from Prof. Scott Farrand, CSUS Math 29B Workshop 25, 26 Spring 2005 and Unified Math 3, G. Rising

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4. Simplify the following. Justify with clear and complete work.

a.  $\frac{\tan^2(x)}{\cos(x)} + \sec(x)$     b.  $\frac{\cos(\alpha)}{1 + \sin(\alpha)} + \tan(\alpha)$     c.  $(\csc(\alpha) - \cot(\alpha))(\sec(\alpha) + 1)$

5. Suppose that  $\sin \alpha = -0.9$ . Find each of the following with using a calculator. Justify with a unit circle drawing.

a. What could be the value for  $\sin\left[\alpha + \left(\frac{\pi}{2}\right)\right]$ ?

b. What could be the value for  $\cos\left[\alpha + \left(\frac{\pi}{2}\right)\right]$ ?

6. Suppose that  $\cos \alpha = 0.2$  and  $\sin \alpha = 0.979796$ , find an angle  $\lambda$  (in terms of  $\alpha$ ) for which  $\sin(\lambda) = 0.2$  and  $\cos(\lambda) = -0.979796$ .

7. Without using your calculator, find all solutions for  $t$  such that  $\sec^2(t) = \frac{4}{3}$ .

8. Without using your calculator, find exact values for the variable between  $0$  and  $2\pi$  for each of the following. Justify with clear and complete work. Verify your solutions with a graph (yea-- now use your calculator).

a.  $\tan(\theta) = \sqrt{3}$     b.  $\cos(\alpha) + 2 = 3\cos(\alpha)$     c.  $2\sin(\theta) + 5 = 4\sin(\theta) + 6$

d.  $3\cot^2(\theta) = 1$     e.  $3\sec(\theta) = -2\sqrt{3}$

9. Find exact numerical values for the following without using your calculator.

a.  $\log_5(125\sqrt{5})$     b.  $\log_{\left(\frac{1}{8}\right)}(16)$     c.  $\log_{\sqrt{b}}(\sqrt[3]{b^2})$

d.  $\log_{\left(\frac{1}{9}\right)}\left(\frac{27}{\sqrt{3}}\right)$     e.  $\log_{\left(\frac{1}{\sqrt{b}}\right)}(b^3)$     f.  $\ln\left((\sqrt{e})^{\ln(e^2)}\right)$

10. Solve for  $x$ :

a.  $\log_x(2x^3 + x^2 - 16) = 2$     b.  $\log_3(x^2 + 4x + 1) - \log_3(x^2 - 5) = 1$