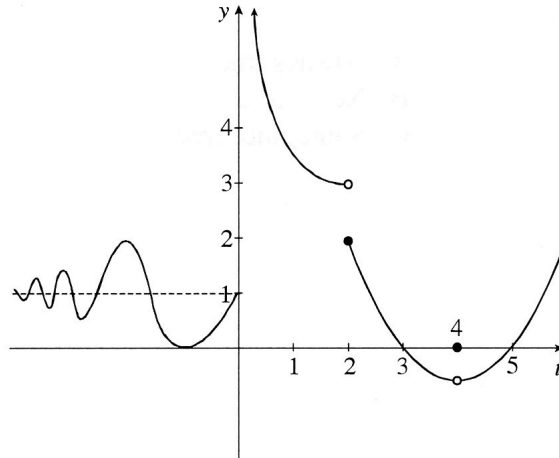


Intro to Calculus Test 8 Mr. Holcomb 2008/2009
Best place for dessert in Santa Cruz?

Problem

1. (8 points) Consider the following graph of the function f .



- a. What is $\lim_{x \rightarrow 0^+} f(t)$? _____
- b. What is $\lim_{x \rightarrow 0^-} f(t)$? _____
- c. What is $\lim_{x \rightarrow 2^-} f(t)$? _____
- d. For what values of x does $\lim_{t \rightarrow x} f(t)$ not exist?

2. (8 points) Let

$$f(x) = \begin{cases} \sqrt{3-x}, & x \leq 1 \\ x^2, & 1 < x < 3 \\ \frac{27}{x}, & x \geq 3 \end{cases}$$

Evaluate each limit, if it exists. If the limit does not exist, explain why not.

- a. $\lim_{x \rightarrow 1^-} f(x)$ b. $\lim_{x \rightarrow 1^+} f(x)$ c. $\lim_{x \rightarrow 1} f(x)$ d. $\lim_{x \rightarrow 3^-} f(x)$
- e. $\lim_{x \rightarrow 3^+} f(x)$ f. $\lim_{x \rightarrow 3} f(x)$ g. $\lim_{x \rightarrow 9} f(x)$ h. $\lim_{x \rightarrow -6} f(x)$

3. (12 points) Let $f(x) = 2x^2 - 5x + 3$.

a. Find the average rate of change of f between $x = 2$ and $x = 2.1$.

b. Find an expression for the slope of the line (the derivative) between the points $(2, f(2))$ and $(2 + h, f(2 + h))$. Simplify your expression.

c. Call your expression from part (b) $g(x)$. Evaluate $\lim_{h \rightarrow 0} g(x)$.

d. What does your answer from part (c) mean in terms of the function f ?

4. (18 points) Evaluate the limits. Justify with clear and complete work.

a. $\lim_{x \rightarrow -4} f(x)$ where $f(x) = \frac{\frac{1}{4} + \frac{1}{x}}{4+x}$

b. $\lim_{x \rightarrow 0} f(x)$ where $f(x) = \frac{\sqrt{3+x} - \sqrt{3}}{x}$

c. $\lim_{x \rightarrow 0} f(x)$ where $f(x) = \frac{\sin(4x)}{x}$

5. (12 points) Evaluate the following limits.

a. $\lim_{x \rightarrow -\infty} f(x)$ where $f(x) = \frac{3x^3 - x + 5}{x^2 - 4}$

b. $\lim_{x \rightarrow +\infty} f(x)$ where $f(x) = \frac{x^2 - \sin(x)}{2x^2 + \cos(x)}$

c. $\lim_{x \rightarrow +\infty} f(x)$ where $f(x) = \frac{2 \log(7^x)}{(x+5)^3}$

6. (5 points) Convert the rectangular (Cartesian) equation $y^2 = 4x$ into a polar equation in the form $r = f(\theta)$.

.

7. (5 points) Convert the polar equation $r = \sec(\theta)$ into a rectangular (Cartesian) equation.

.

8. (7 points) Sketch the graph of a function $f(x)$ for which the following are all true:

$$\lim_{x \rightarrow -3} f(x) = +\infty$$

$$\lim_{x \rightarrow -1} f(x) = 2$$

$$f(-1) = -2$$

$$\lim_{x \rightarrow 0^-} f(x) = 0$$

$$\lim_{x \rightarrow 0^+} f(x) = -1$$

$$\lim_{x \rightarrow 2} f(x) \text{ does not exist}$$

$$\lim_{x \rightarrow 3} f(x) \text{ does not exist, but } \lim_{x \rightarrow 3^-} f(x) \text{ and } \lim_{x \rightarrow 3^+} f(x) \text{ do exist.}$$