Name ________________ Date _______ Class # ______ Block ______

Intro to Calculus

Polar Tic-Tac-Toe[1]

Goals

I can locate points using the Polar Coordinate System.

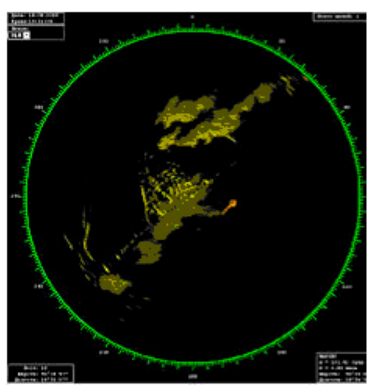
I can use a graphing calculator to graph polar functions.

I can analyze, generalize, and describe patterns related to polar coordinates and functions.

Storms are the stuff of legends. Unfortunately, those who achieve heroism against natures fury often pay the ultimate price. Their efforts are chronicled in many ways— from a song about 15-year old Hazel Miner, who died protecting her younger brother from a North Dakota spring blizzard, to a best-selling book about the Andrea Gail.

Efforts to predict, prepare for, avoid, and survive storms in the present day depend on radar, a technology that originated in the 1930's when British scientists discovered that radio waves bounce off objects and a portion of the wave returns towards the source. Knowing the time interval when the signal is sent and subsequently returned allows one to determine how far away the object is. In 1968, U.S. Air Force scientists found a way to use radar to depict precipitation. Today, radar is routinely used by meteorologists to track storms, and by ships at sea to avoid collisions with land, icebergs, and other ships, especially during foggy weather.

Figure 1



Example of a radar screen.

Radar operates by sending special radio signals outward from a central location such as a weather observatory or a ship. The signal's return is translated into information about the distance and direction to the objects such as moisture-bearing clouds or ships.

Since distance and direction are radar's fundamental units, its coordinate system is unlike the rectangular system you have grown use to. Rectangular coordinates locate a point by specifying distance in two directions— one horizontal (x-value), the other vertical (y-value)— from a reference point often referred to as the "origin" which is given coordinate values of (0,0). In contrast, the polar coordinate system locates a point by specifying a single distance and a direction from the reference point.

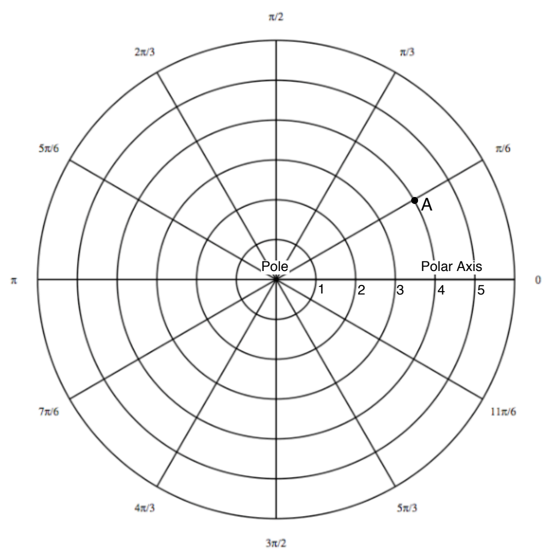
The first person to contemplate polar coordinates was probably Isaac Newton (1642-1727), but it was Jakob Bernoulli (1654-1705), who proposed the system in 1691. However, the writings on Leonhard Euler (1707-1783) around 1750 made polar coordinates popular. First used to treat certain calculus problems, the system is far more important today than its originators could have imagined.

Since polar coordinates area based on distance and direction, polar graph paper shows distances from a reference point, called the pole, as concentric circles and directions from a reference line, called the polar axis, as segments radiating outward from the

[1] Adapted from "Precalculus: Modeling our World", CoMap Preliminary Addition pp.346-353

pole. The graph below (Figure 2) shows six units of distance, and direction in increments of 30˚. (To avoid clutter, polar graph paper that uses finer increments of direction often omits some or all direction lines near the pole.)

Figure 2



It is customary to associate a point with an ordered pair in which the distance is written first and the direction second. For example, the pair $\left(4, \frac{\pi}{6}\right)$. Unlike rectangular coordinates, the coordinates of a point in the polar system are not unique because angles can have measures greater than 2π (360˚ radians) or less than 0 and distance can be negative. Point *A* in the graph for example is also a location specified by $\left(4, \frac{13\pi}{6}\right)$, $\left(4, \frac{-11\pi}{6}\right)$, and $\left(-4, \frac{7\pi}{6}\right)$ as well as by an infinite number of other ordered pairs.

To acquaint yourself with this coordinate system, you will play polar tic-tac-toe.

The game is played by two people or by two teams of people using a piece of polar graph paper as the game board. Here are the rules:

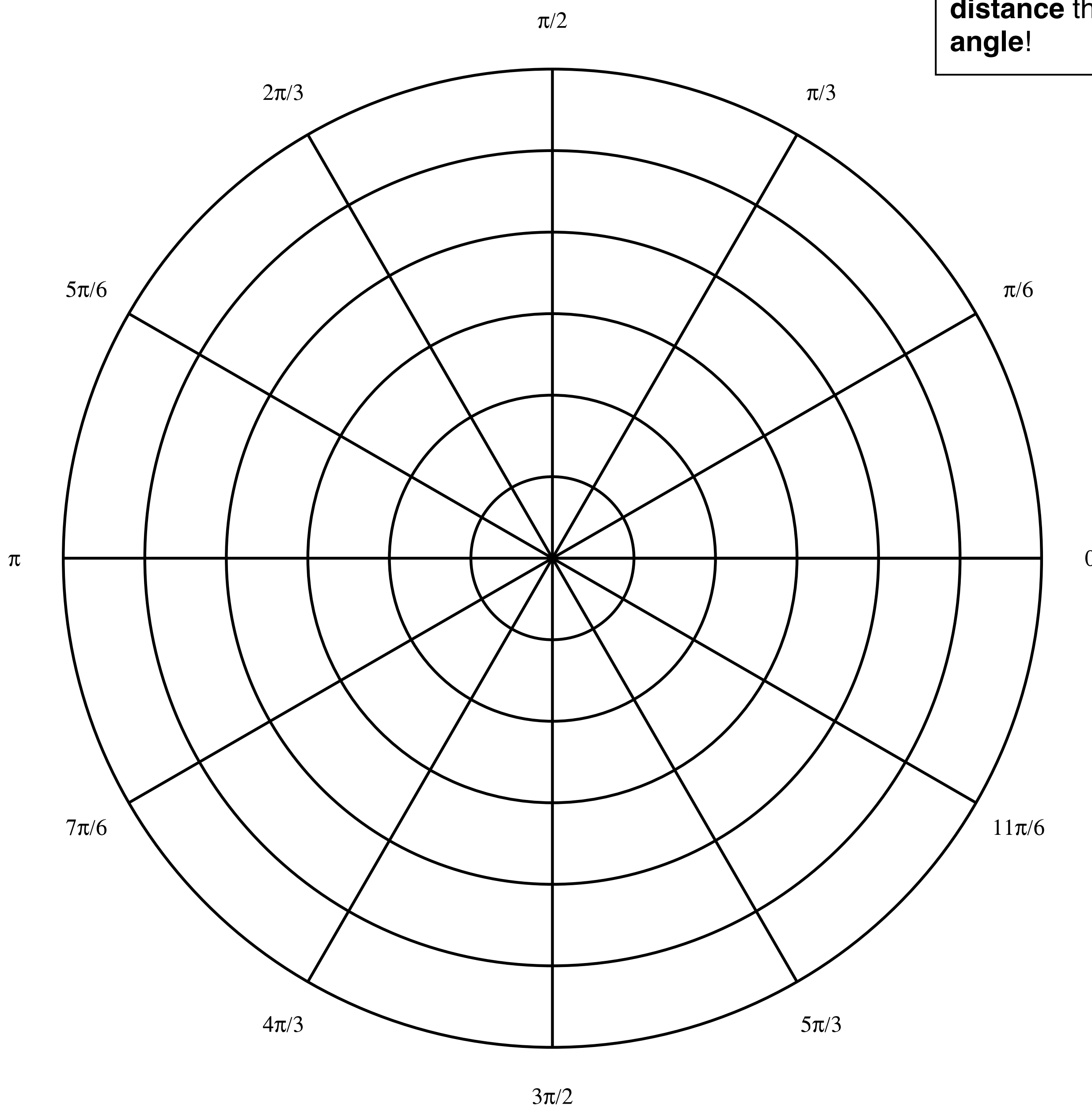
1. Decide on who is going to make the first move.
2. The first person specifies the location of any intersection of a circle and a ray (with the exception of the pole) by calling out **first a distance** from the pole and **then an angle** of rotation.
3. The second person marks the corresponding point with an "X". If the players agree on the location of the point, the caller and the marker switch roles, with the new marker using "O", instead of an "X".
4. If the players disagree about the location of the point, the disagreement is resolved by players (or, if necessary, by consultation with an outside authority). If the person who called the point is wrong, the point is erased and the second person becomes the caller. If the person who marked the point is wrong, the mark is erased and the correct point is marked. Then the caller gets another turn calling a point.
5. The caller wins when the marker is forced to complete **four adjacent** marks along one of the following: **a directional ray; a circle, a spiral** (clockwise or counterclockwise).

Play the game several times against one or more opponents. As you play the game, in addition to labeling the point with an "X" or an "O", also record the ordered pair (distance, angle). Remember the main goal for playing this game is to become fluent with the polar coordinate system.

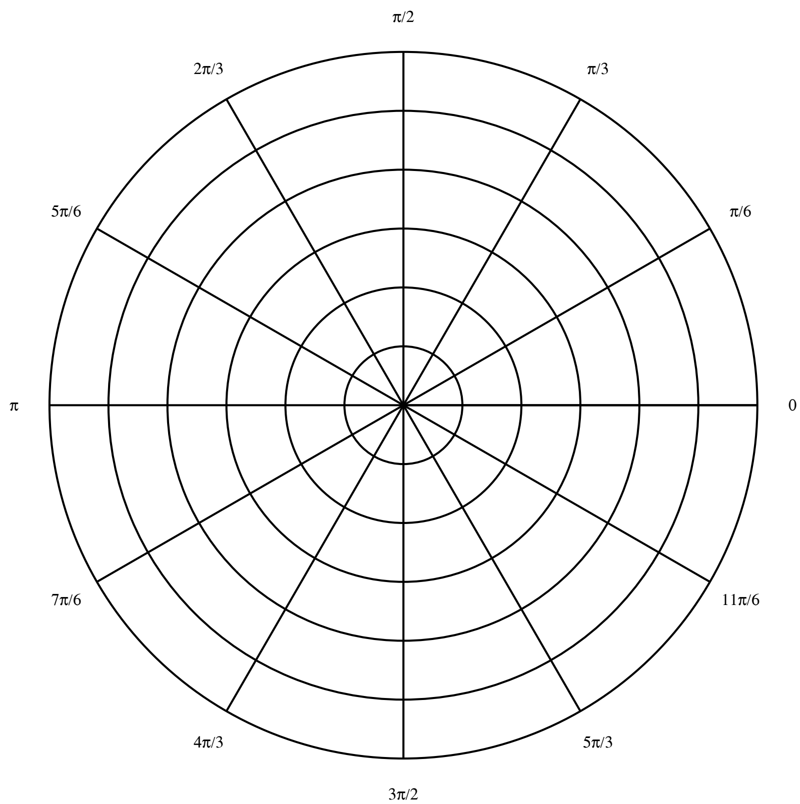
Intro to Calculus: Polar Tic-Tac-Toe

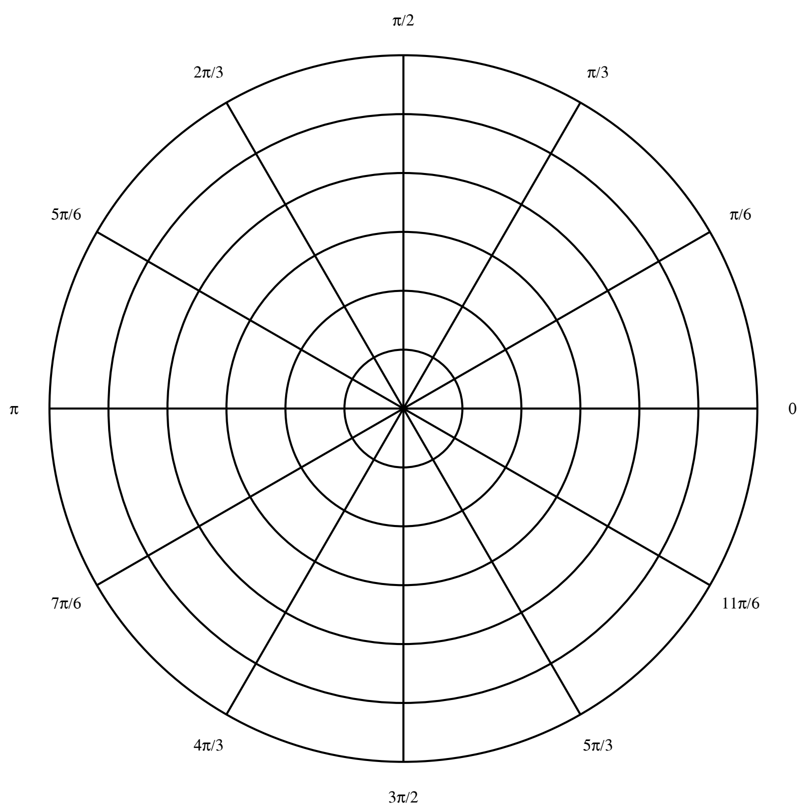
<u>Remember</u>

Call out first a **distance** then an **angle**!



Intro to Calculus: Polar Tic-Tac-Toe





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Intro to Calculus

Polar Tic-Tac-Toe: Problems

Goals

I can locate points using the Polar Coordinate System.

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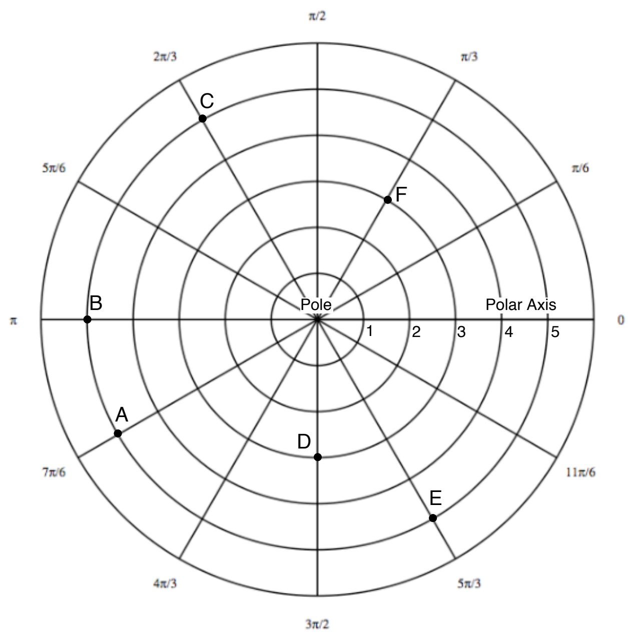
1. If possible, match the coordinates to the point they represent.

   $\left(5,\frac{2\pi}{3}\right)$, $(-5,\pi)$, $\left(-3,\frac{4\pi}{3}\right)$, $(-5,0)$, $\left(5,\frac{31\pi}{6}\right)$, $\left(-3,\frac{-11\pi}{2}\right)$, $\left(4,\frac{5\pi}{3}\right)$



2. What do all polar pairs that represent the pole have in common?

3. Points *A* and *B* are coincident but there coordinates are not identical. If point *A* is located at $\left(3,\frac{5\pi}{6}\right)$, what could be another coordinate pair for point *B*? What can you say in general about the coordinates of point *B*?

4. Points *A* and *B* are coincident but there coordinates are not identical. In fact their angles differ by π radians. What has to be true about the distance coordinates of points *A* and *B*?

Polar Tic-Tac-Toe: Problems

5. a. Suppose you won at polar tic-tac-toe by getting 4 adjacent points along a directional ray. What has to be true about the coordinates of these 4 points?

   b. Suppose you had won by getting the 4 adjacent points to be in a circle. What gas to be true about the coordinates of these 4 points?

   c. What about if you had won by making a clockwise spiral. What has to be true about the coordinates of the points?

6. Consider the polar equation $r = \sin(\theta)$. Calculate the following values and then plot them using a polar graph.

Table for Problem 6

r	θ	r	θ	r	θ
0	0		$\frac{2\pi}{3}$		$\frac{4\pi}{3}$
1	$\frac{\pi}{6}$		$\frac{5\pi}{6}$		$\frac{3\pi}{2}$
	$\frac{\pi}{3}$	-1	π		$\frac{5\pi}{3}$
	$\frac{\pi}{2}$		$\frac{7\pi}{6}$		$\frac{11\pi}{6}$

7. A student won a game of polar tic-tac-toe by getting four adjacent points along a circle of radius 3 such that the first point was located at $\left(3, \frac{2\pi}{3}\right)$ and the last point was located at $\left(3, \frac{7\pi}{6}\right)$.

   a. What are the coordinates of the other points?

   b. Create a polar equation for the radius as a function of the angle.

8. A student won a game of polar tic-tac-toe by getting four adjacent points along a directional ray. The first point was at $\left(1, \frac{5\pi}{6}\right)$ and the last point was at $\left(-4, \frac{11\pi}{6}\right)$. Create a polar equation for the radius as a function of the angle.

   a. What are the coordinates of the other points?

   b. Create a polar equation for the radius as a function of the angle.

9. A student won a game of polar tic-tac-toe by getting four adjacent points along a counterclockwise spiral. The first point was at $(1,0)$ and the last point was at $\left(4, \frac{\pi}{2}\right)$.

   a. What are the coordinates of the other points?

   b. Create a polar equation for the radius as a function of the angle.

Polar Tic-Tac-Toe: Problems

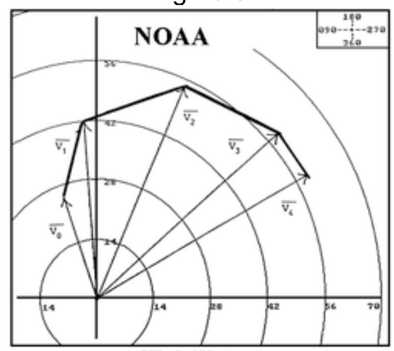
10. A student won a game of polar tic-tac-toe by getting four adjacent points along a clockwise spiral. The first point was at $\left(2, \frac{5\pi}{3}\right)$ and the last point was at $\left(5, \frac{7\pi}{6}\right)$.

    a. What are the coordinates of the other points?

    b. Create a polar equation for the radius as a function of the angle.

11. A hodograph (Figure 3) is a polar plot of wind speed and direction (in which the wind is traveling) at various altitudes. **(Direction is measured clockwise from north.)** Measurements are taken at uniform increments of altitude, the pairs (velocity, direction) plotted, and the points connected in order from the lowest altitude to the highest. The plot that results is used to forecast the type of thunderstorms that may develop and the potential for tornadoes and other severe weather.

Figure 3



Wind

Height (km)	Direction (Degrees)	Speed (Km/Hr)
0	161	25
1	175	42
2	204	54
3	225	59
4	241	59

Hodograph of upper air winds (Source Wikipedia)

    a. The table below displays hypothetical wind data. Construct a hodograph.

Altitude (meters)	Velocity (mph)	Direction (degrees)
1000	13	282
2000	21	265
3000	34	243
4000	32	261

Altitude (meters)	Velocity (mph)	Direction (degrees)
6000	36	198
7000	38	185
8000	41	170
9000	51	153

    b. Certain patterns in hodographs— spirals and loops, for example— indicate a potential for severe weather, especially when accompanied by certain other conditions. Describe any patterns you see in the hodograph you created.

    c. Suppose you were riding in a balloon that had been launched from ground level and went straight up. Write a description of your trip between 0 and 9000 meters based on the data and your hodograph.

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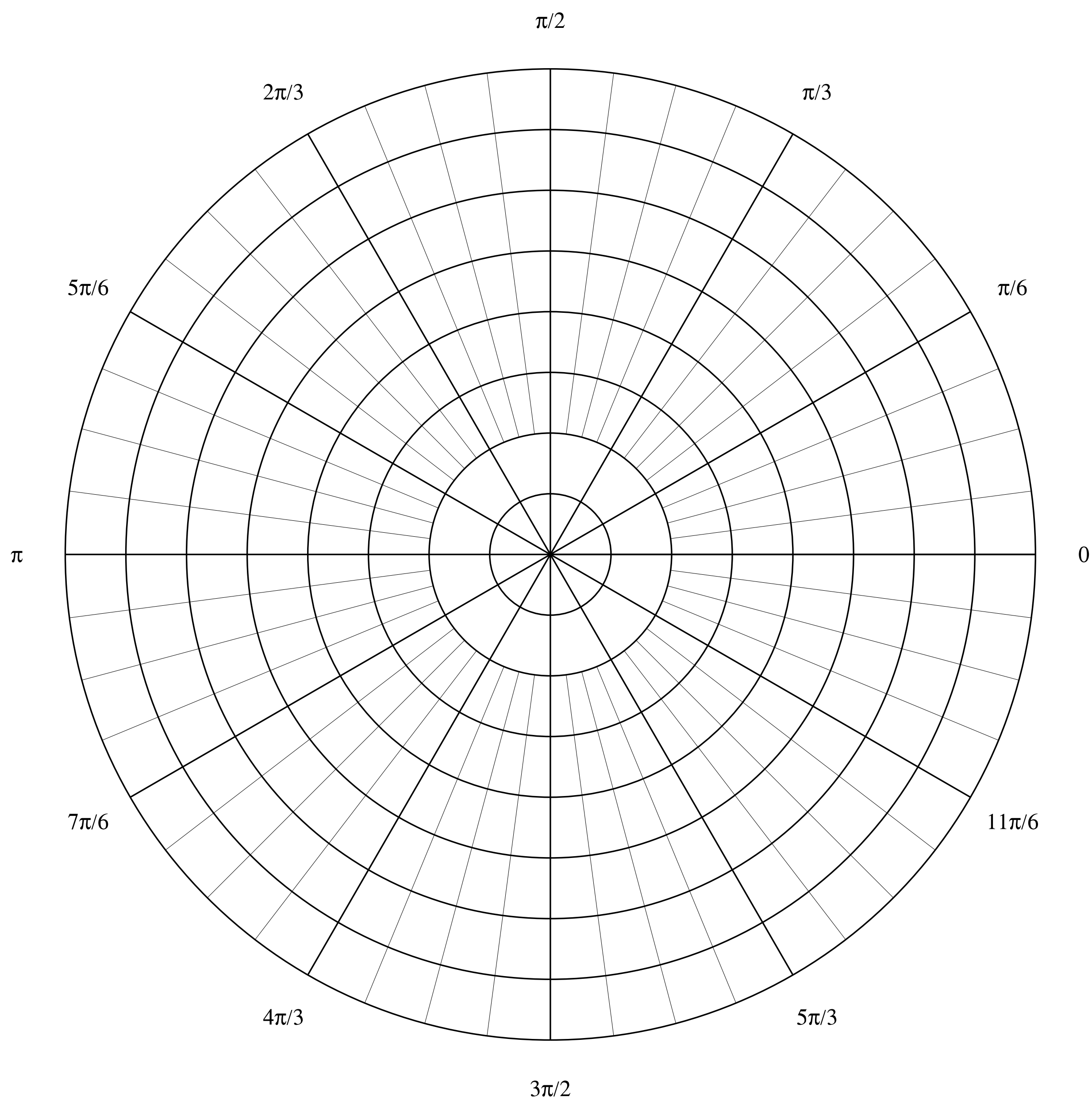

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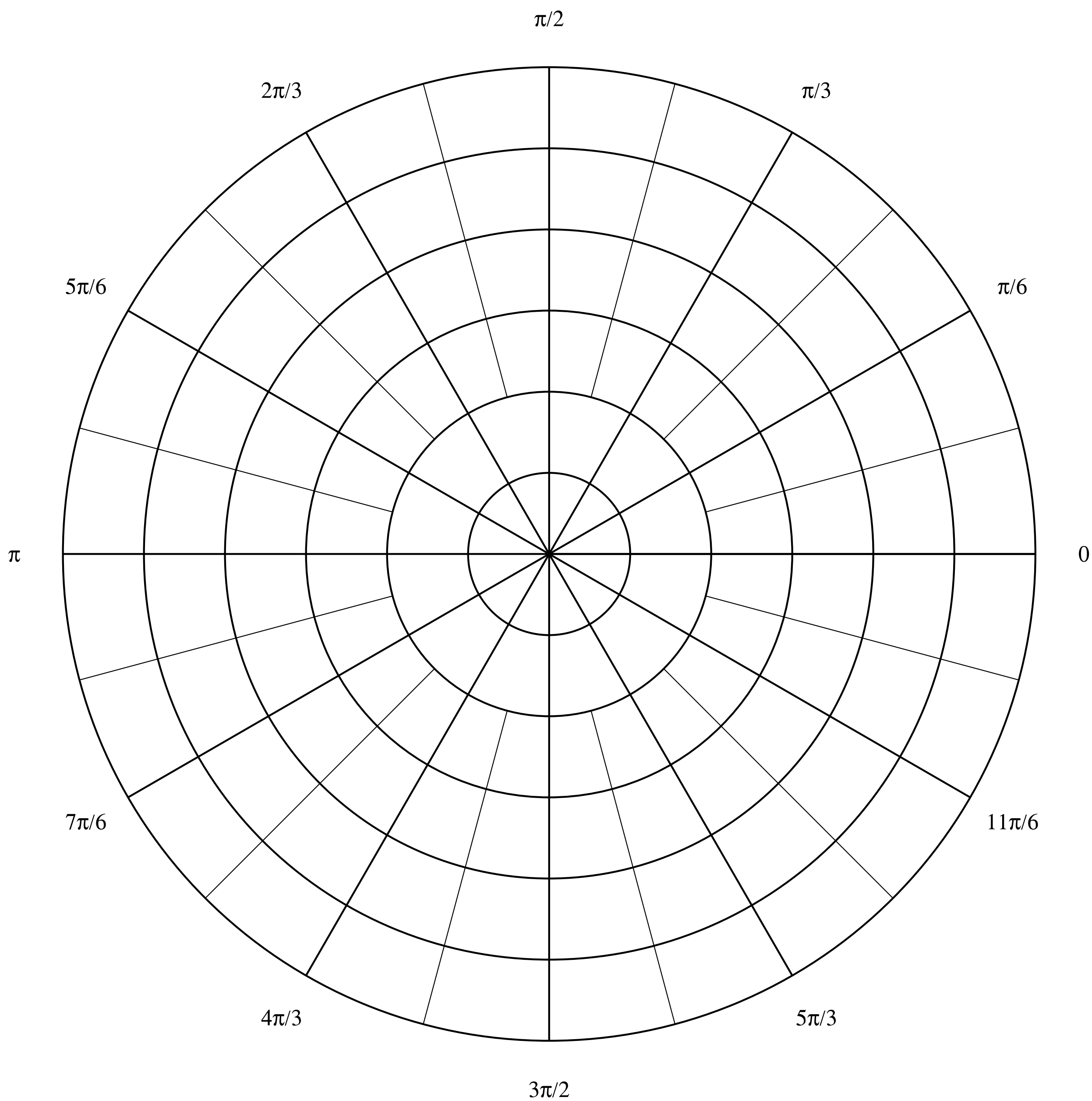

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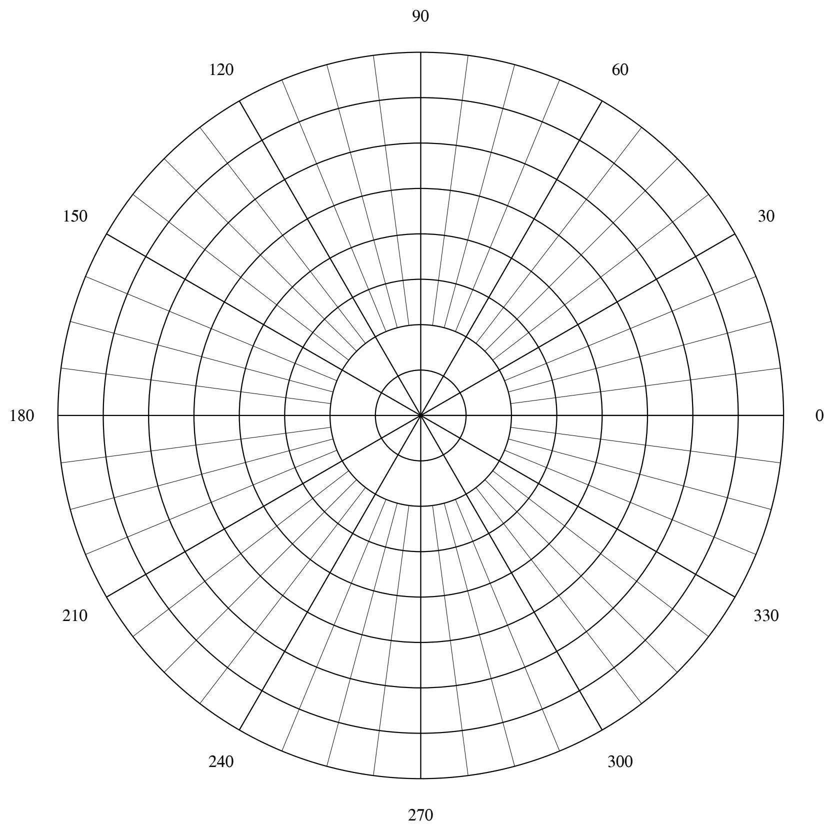

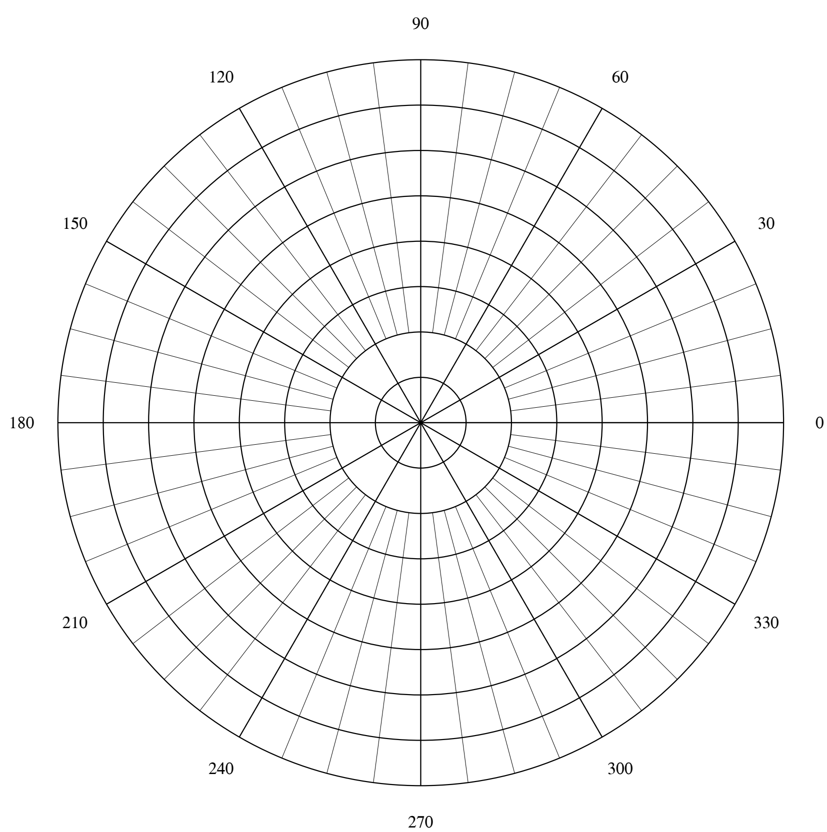

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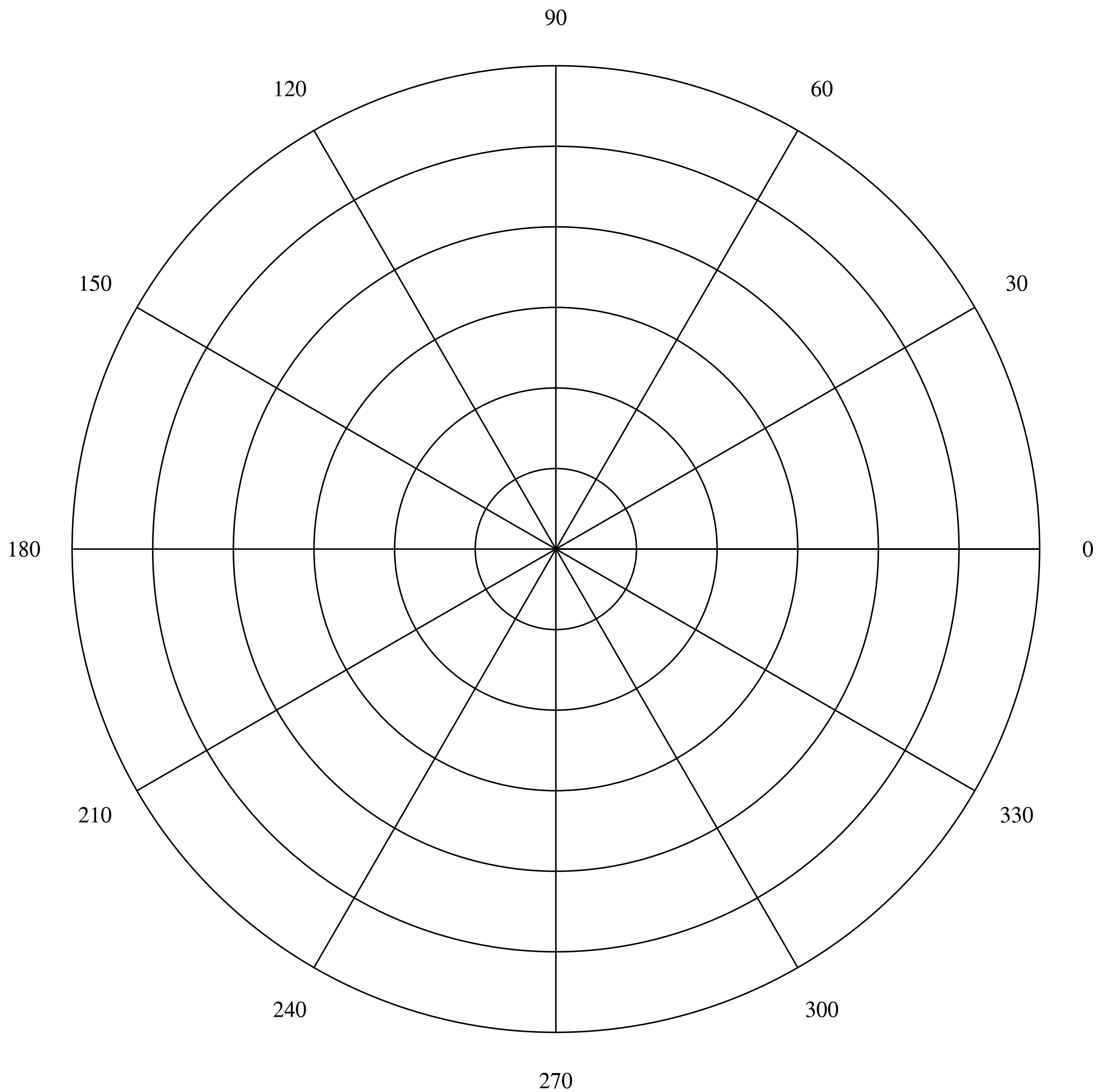

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