

Intro to Calculus

Homework 8¹

Goals

I can express one function in terms of transformations of another function.

I can describe a sequence of transformations which will transform the graph of one function into the graph of another.

I can describe the graph of a function in terms of transformations of one of the core functions.

1. Express the function g in terms of the function f .

a. $f(x) = x^3 - 2x$; $g(x) = (x + 4)^3 - 2(x + 4)$

b. $f(x) = x^3 - 2x$; $g(x) = x^3 - 2x + 6$

c. $f(x) = x^3 - 2x$; $g(x) = 5x^3 - 10x$

d. $f(x) = x^3 - 2x$; $g(x) = 8x^3 - 4x$

e. $f(x) = x^3 - 2x$; $g(x) = 2x - x^3 - 7$

f. $f(x) = \sqrt{4x^2 + 7}$; $g(x) = \sqrt{36x^2 + 7}$

g. $f(x) = \sqrt{4x^2 + 7}$; $g(x) = \frac{3}{4}\sqrt{4x^2 + 7} - 12$

2. Go back to problem 1. For each, describe a sequence of transformations that will transform the graph of the function f into the graph of the function g .

3. Let $f(x) = x^3 - 2x$, and let $g(x)$ represent this function after the transformations described below. Express $g(x)$ in terms of $f(x)$.

a. Move f two units to the right and up 4 units.

b. Stretch f two units in the vertical direction and then move it two units to the left.

c. Reflect f over the x -axis, move it two units down.

d. Compress f by a factor of two in the horizontal direction, move it down two units.

¹ This work is based on material from Prof. Scott Farrand, CSU Sacramento

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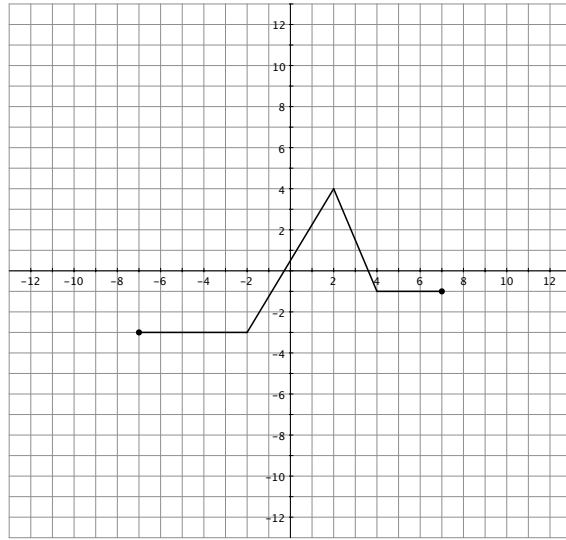
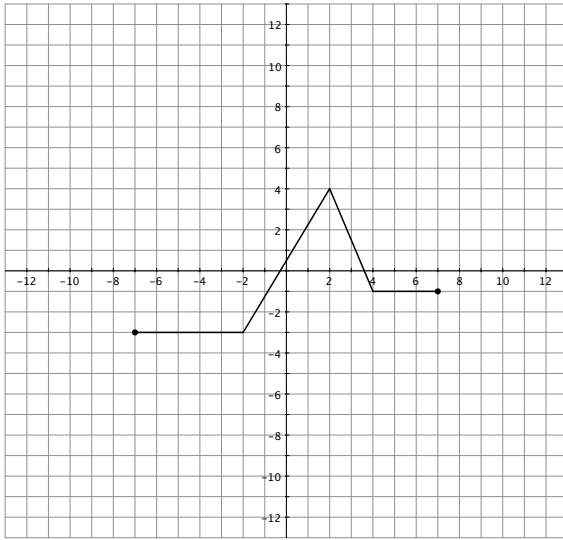
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4. The graph of function f is given below. For each function g below, list a *sequence* of transformations that transform the graph of f into the graph of g , and use these to sketch the graph of $y = g(x)$.

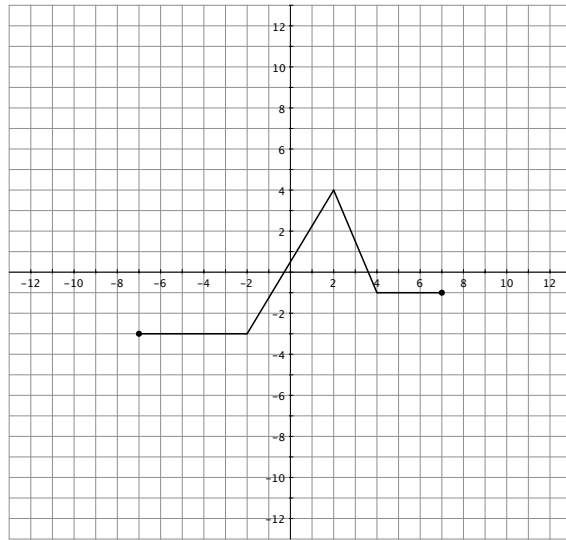
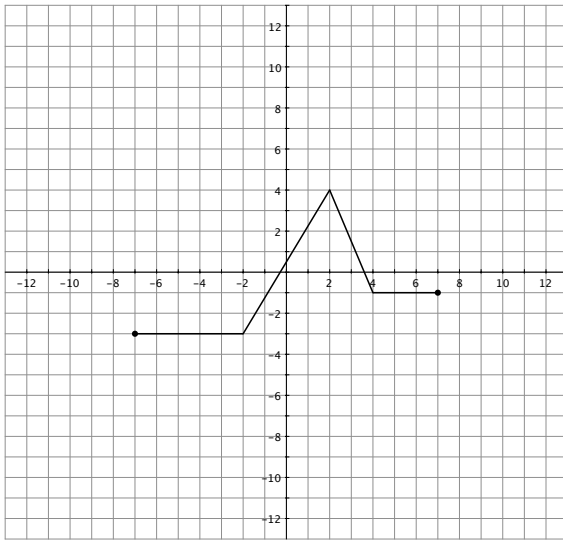
a. $g(x) = f(x+2) - 6$

b. $g(x) = 3f(-x)$



c. $g(x) = -2f(x-1) + 3$

d. $g(x) = -2f(-x) - 1$



1. A function f is given with a sequence of transformations. The graph of a new function g is obtained from the graph of f by performing these transformations (in the order given). Write g in terms of f and use this to find the rule of g .

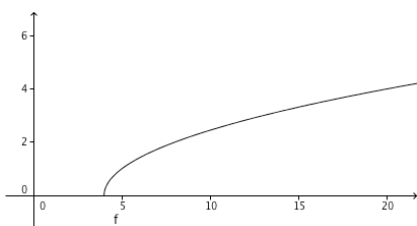
- a. $f(x) = 3x^2 - 4$; shift the graph horizontally 7 units to the right and then shift it vertically downward 5 units.
- b. $f(x) = x^2 + 2x - 1$; reflect the graph across the y -axis, vertically shift upward 3 units, and reflect the graph across the x -axis.
- c. $f(x) = \sqrt{x+2}$; compress the graph toward the y -axis by a factor of 2, shift the graph horizontally 3 units to the left, compress it toward the x -axis by a factor of 4, and then reflect it about the x -axis.
- d. $f(x) = \sqrt{-x}$; reflect the graph across the y -axis, vertically shift 6 units downward, compress toward the x -axis by a factor of 2, and reflect across the x -axis.

6. We call the following functions core functions because, using transformation, we can build many other functions from them.

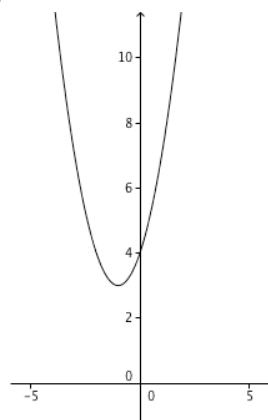
$$f(x) = x^2, \quad g(x) = x^3, \quad h(x) = \sqrt{x}, \quad j(x) = |x|, \quad k(x) = \sqrt{1-x^2}$$

- a. Roughly sketch each of the core graphs. Use your grapher to help.
- b. Each of the graphs below is obtained from one of these core graphs using a transformation or sequence of transformations. Express the graph in terms of one of the core functions, and then use this to find an algebraic rule for the function shown. Feel free to check your answers with a grapher.

(1)



(2)



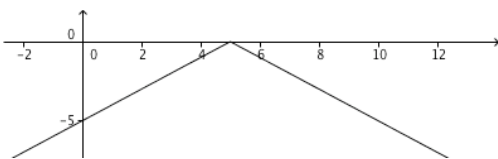
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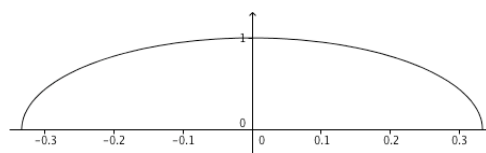
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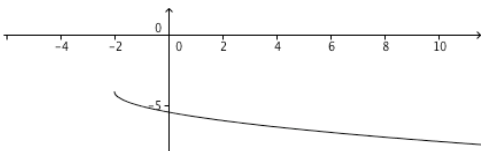
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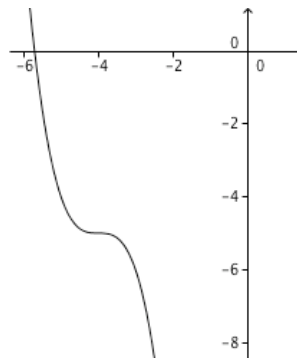
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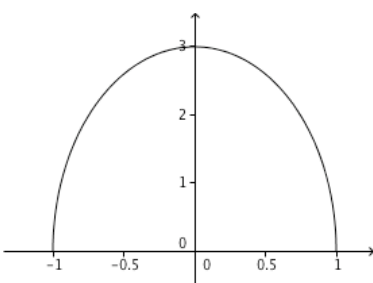
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(7)



(5)



7. Each function $v(x)$ is obtained from one of the following core functions:

$$f(x) = x^2, \quad h(x) = \sqrt{x}, \quad k(x) = \sqrt{1-x^2}$$

Write $v(x)$ in terms of one of these core functions. Use this to list the transformations needed to transform the graph of the core function into the graph of $y = v(x)$ and then sketch a graph of $y = v(x)$.

a. $v(x) = (x+3)^2$

d. $v(x) = \sqrt{1-16x^2}$

b. $v(x) = -\sqrt{x-7}$

e. $v(x) = -\sqrt{16-16x^2}$

c. $v(x) = \sqrt{-x} + 7$