Name _____ Date _____ Class # _____ Block _____

Intro to Calculus

Lesson for Homework 29

In this lesson you will be introduced to a fourth main topic of Calculus, continuity, and will see how it can help you solve each of the puzzles posed above. But before we dive in, discuss each puzzle with your group and see if you can reach a consensus on the solutions.

Goals

I can explain what it means for a function to be continuous.

I can determine if a function is continuous.

I can explain the meaning of the Intermediate Value Theorem and can apply it solve problems.

Puzzler 1

A monk leaves at 8AM on day one from his monastery, at the foot of a tall mountain, and follows a trail to the summit, which he reaches at 8PM on the same day, after 12 hours spent walking, looking at the landscape, or sitting in prayer. On the following day, the monk leaves the summit at 8AM and follows the same trail down to reach his monastery at 8PM, again after 12 hours.

The monk has made frequent stops on his way up and down the mountain. In one occasion during his ascent he even walked back down the trail for a while, to retrieve a cellphone he had left behind.

The question: Is there at least one point along the trail where the monk has been at the same time of the day, on day one and on day two? Justify.

Intro to Calculus: Lesson for Homework 29— Continuity and the Intermediate Value Theorem

1. So far you have been introduced to three main topics of Calculus. What are they?

2. Suppose that $f(1) = -5$ and $f(3) = 5$.

   Does there have to be a value of $x \in (1,3)$ such that $f(x) = 0$? Justify.

3. Suppose that some function, *f*, is continuous over some interval. Intuitively what does this mean?

4. How could a function fail to be continuous? Draw examples of all the "failures" you can think of.

5. Consider some arbitrary function, *f*, that is defined on the domain $x \in [a,b]$.

   a. What would have to be true about *f* in order for it to be continuous at some interior point, $x = c$?

   b. What would have to be true about *f* in order for it to be continuous at its left endpoint? At its right endpoint?

   c. Look back at your answer to question 4. With regards to your answers to (a) and (b) above, what's wrong with each of them? Label them with their "problems".

6. Consider the function below

$$h(x)=\begin{cases} cx^2,\ x<1 \\ 4,\ x=1 \\ -x^3+mx,\ x>1 \end{cases}$$

Find values of c and m would make this function continuous at $x=1$, or explain why such values do not exist.

7. Consider the function $f(x)=\dfrac{x^3-7x-6}{x^2-9}$.

a. Why $f(x)$ not continuous? Justify.

b. If possible, fix the discontinuity. If not, explain why not.

8. Algebraically speaking, what could cause a function to fail at being continuous?

9. What types of functions do you think are continuous over their domain? Why?

10. Trick question: Is $y=\tan(x)$ continuous?

11. Use limits and what it means for a function to be continuous at a point to show that $f(x)=x^2+\sqrt{7-x}$ is continuous at $x=4$.

Intro to Calculus: Lesson for Homework 29— Continuity and the Intermediate Value Theorem

An Implication of Continuity

12. Imagine some function, $y = f(x)$, which is continuous on some interval $x \in [a,b]$. Now pick a value between $f(a)$ and $f(b)$ -- call it $f(c)$. Question: does *c* have to be between *a* and *b*? Justify.

13. Complete the following:

    A function $y = f(x)$ that is _____ on a closed interval $[a,b]$ takes on _____ value between $f(a)$ and ____.

14. In "non-mathy" language, explain what the Intermediate Value Theorem means.

15. Why does the function $f(x) = 4x^3 - 6x^2 + 3x - 2$ has to have an x-intercept on $x \in [1,2]$.

16. Solve Puzzler 1 using the Intermediate Value Theorem.