Name ________________ Date _______ Class # ______ Block ______

Intro to Calculus

Homework 29— Continuity and the IVT

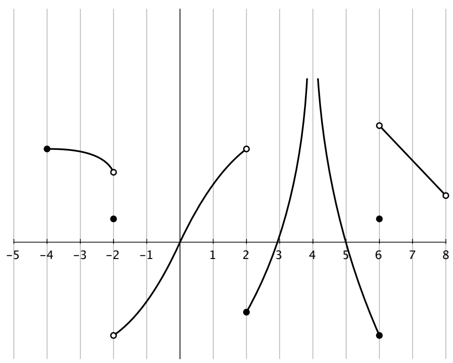
Goals

I can explain what it means for a function to be continuous.

I can determine if a function is continuous.

I can explain and use the Intermediate Value Theorem.

1. From the graph of the function *g* below, state the intervals on which *g* is continuous.



2. Which of the functions described below represent continuous functions. State the domain, if possible, for each example.

   a. The temperature on a specific day at a given location considered as a function of time.

   b. The number of stamps it takes to mail a package considered as a function of the weight of the package.

   c. The number of unemployed people in the United States during 2008 considered as a function of time.

3. a. Create your own description of a continuous function similar to the ones describe above

   b. Now do one for a function which is not continuous.

4. Find constants a and b so that the given function will be continuous for all x.

   a. $f(x)=\begin{cases} ax+3, & x>5 \\ 8, & x=5 \\ x^2+bx+1, & x<5 \end{cases}$

   b. $f(t)=\begin{cases} \dfrac{at-4}{t-2}, & t\neq 2 \\ b, & t=2 \end{cases}$

5. a. Write $f(x)=\sin\left(x^2\right)$ as the composition of two functions, g, and h.

   b. Sketch a graph the includes each of the three functions on $x\in[-\pi,\pi]$.

   c. On what interval do you suspect $f(x)$ to be continuous?

6. Repeat the previous question letting $f(x)=\dfrac{1}{\sqrt{x^2+7}-4}$.

7. In general, what can you say about the continuity of a composition of two functions?

8. Use the Intermediate Value Theorem to show that there is a root on the given equation in the specified interval.

   a. $x^4+x-3=0,\ (1,2)$ b. $\cos(x)=x,\ (0,1)$

9. True or False. If false, give a counter-example.[1]

   a. If $f(-1)=-1$ and $f(1)=1$, then $f(0)=0$

   b. If $f(-1)=-1$ and $f(1)=1$, then there is a point c such that $-1<c<1$ and $f(c)=0$.

   c. If $f(-1)=-1$ and $f(1)=1$ and $f(x)$ is continuous, then there is a point c, $-1<c<1$, such that $f(c)=0$.

   d. If $f(0)=0$, $f(1)=10$ and $f(x)$, then on the interval $[0,10]$, $f(x)$ must have a maximum or a minimum value.

10. Find the discontinuities of $f(x)=\begin{cases}\dfrac{x^2-1}{x^2+x}, & x>-3\\ 1, & x\le -3\end{cases}$.

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[1] Better File Cabinet Problem utf07-001

11. Find the discontinuities of $f(x)=\begin{cases} \dfrac{x}{x^2+1}, & x>-1 \\ \dfrac{x+1}{x^2-1}, & x<-1 \\ \dfrac{1}{2}, & x=-1 \end{cases}$.

12. Let $f(x)$ be a continuous function. Then there is (sometimes, always, never) a value c such that $f(c)=c$. Explain.

13. Multiple Choice: The function f is continuous on the closed interval $[1,4]$ and has values that are given in the table below:

x	1	2	3	4
$f(x)$	3	4	a	2

The equation $f(x)=1$ must have at least two solutions in the interval $[1,4]$ if $a=$

(A) 0 (B) 1 (C) 2 (D) 5 (E) y --- answer is (A)

14. For each of the following functions find an integer n so that $f(x)=0$ for some x between n and $n+1$.[2]

a. $f(x)=x^3-x+3$

b. $f(x)=x^5+x+1$

c. $f(x)=4x^2-4x+1$

d. $f(x)=x^5+5x^4+2x+1$

15. a. Explain why $f(x)=x^5+10x^3+45x-10$ has exactly one real root. Find the integer closest to it.[3]

b. Explain why $f(x)=x^5-10x^3+45x-10$ has exactly one real root. Find the integer closest to it.

c. Create a seventh degree polynomial that has exactly one real root.

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[2] Better File Cabinet, UTF19-008

[3] Better File Cabinet, UTF16-003