Name ________________ Date _______ Class # ______ Block ______

# Intro to Calculus

## Lesson for HW 27- Know Your Limits[1]

**Goals**

- I can explain the essential problem addressed by the concept of limits.
- I can solve a limit problem using a graph, a table of values, or algebra.

**Why Limits?**

The work you have done so far has allowed you to see two important concepts of calculus in action: the derivative and the definite integral.

1. Explain the ideas behind the concepts of the derivative of a function and the definite integral from three different perspectives:

   a. In terms of the graph of a function.

   b. In terms of the numerical values of a function.

   c. In terms of some sort of physical event.

   d. In terms of a process of a limit.

As you have seen, calculus changes, or at least refines, our perception of motion and while it provides us with a new tool for solving problems it also raises some additional questions.

2. What are some mathematical issues that our work with calculus has raised?

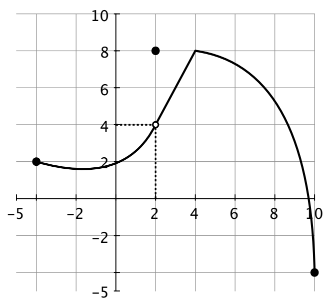
**What are limits from an intuitive perspective?**

As you have seen, a key question is how to deal with these quantities which we wish to become in effect equal to zero without actually becoming zero. Indeed a subtle and perplexing problem. In this lesson we will appeal to our intuition in order to find a solution.

3. What is the essential problem we are trying to solve in our next section of work?

[1] Based on materials from Ithaca College Calculus, Stewart Calculus, Wikipedia, Farrand/Poxon Calculus

Consider the graph shown below. Suppose that you are a bug on the graph walking from the left to the right. Since you are a very well educated bug you obviously would recognize the idea of coordinates!




4. As you get closer and closer to the value $x = 2$, but not equal to $x = 2$, what does the $y$-value get closer and closer to?

5. Why are we being so careful about the “not equal to” part?

**What’s the notation? How do we graphically determine a limit of a function?**

This intuitive idea is at the heart of the mathematical concept of a limit. The limit of a function $f(x)$ describes the behavior of the function close to a particular value of $x$. It does not necessarily say anything about the actual value of the function at $x$.

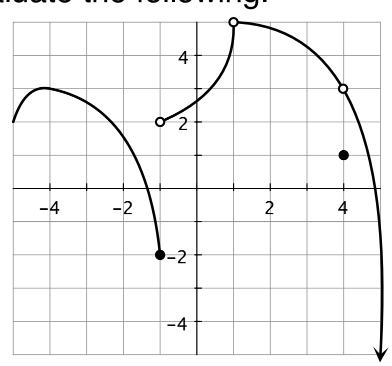
We can express this idea using the following notation

$$\lim_{x \to c} f(x) = L$$

6. State in words how to read the above notation.

7. Show how to express the concept of coming up to a specific $x$ value, $c$, from either the left-side (from below) or from the right side (from above).

8. Using the graph of the function $f$, evaluate the following.[2]




a. $\lim_{x \to -4^-} f(x)$ b. $\lim_{x \to -4^+} f(x)$

c. $\lim_{x \to -4} f(x)$ d. $f(-4)$

e. $\lim_{x \to -2^-} f(x)$ f. $\lim_{x \to -2^+} f(x)$

g. $\lim_{x \to -2} f(x)$ h. $f(-2)$

i. $\lim_{x \to 1} f(x)$ j. $\lim_{x \to 4} f(x)$

k. $\lim_{x \to 5^-} f(x)$

[2] An interactive quiz for the material for this problem can be found here.

9. What do we mean by $\lim_{x \to a} f(x) = +\infty$?[3]

## How can a limit be found algebraically?

10. Solve the following problems. Confirm graphically. Include both a sketch and the algebra.

    a. $\lim_{x \to 3} 2x^2 + 5x - 1$

    b. $\lim_{x \to 3} \frac{x-3}{x^2}$

    c. $\lim_{x \to 3} \frac{x^2}{x-3}$

    d. $\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$

    e. $\lim_{x \to 2} \frac{x-2}{\left(\frac{1}{x-2}\right)}$

    f. $\lim_{t \to 0} \frac{\sqrt{t^2+9}-3}{t^2}$

    g. $\lim_{x \to 0} |x|$

[3] Check out The Math Page for more info and some practice!

h. $\lim_{x \to 0} \frac{(2+x)^3 - 8}{x}$

11. What is meant by the term “indeterminate form”?

12. Describe a general method for calculating the limit of a function.

13. a. Sketch a function, $f$ ,such that $f(x)$ gets as close as you like to 3 as $x$ gets as close as you like to 7, but right at $x$ equals 7 the value of $f(x)$ is -2.

b. Write the above statement using limit notation using “$f(x)$” to represent the function.

c. Create an equation for $f(x)$.

14. a. Sketch a function, $f$, such that $f(x)$ goes to positive infinity as $x$ gets as close as you like to $\frac{\pi}{2}$ from below, and $f(x)$ goes to negative infinity as $x$ gets as close as you like to $\frac{\pi}{2}$ from above.

b. Write the above statement using limit notation using "$f(x)$" to represent the function.

c. Create an equation for $f(x)$.

**Summary**

15. What are the key ideas you should focus on from this lesson?

16. What will you need to practice?

17. What should you clarify?