

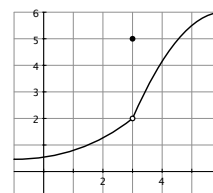
Intro to Calculus

Homework 27

Goals

I can explain what is meant by the term "limit" and can find limits using a graph, a table, or an equation.

- Using a graph of a function, write an explanation which could be used to explain the concept of a limit which a middle school student could understand.
 - What key ideas does a middle schooler really need in to be able to understand your explanation?



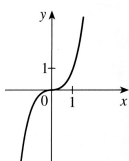
order

- Match each of the following without using a calculator.

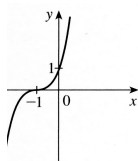
a. $y = \sqrt{x}$ b. $y = x^3$ c. $y = 4 - x^2$ d. $y = (x - 2)^3 + 3$

e. $y = 2\sqrt{x}$ f. $y = (x + 1)^3$ g. $y = -2^x$ h. $y = 1 + x^{-1}$

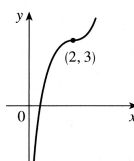
S.



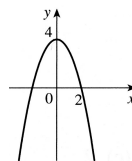
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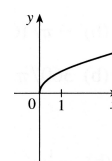
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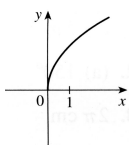
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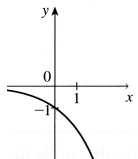
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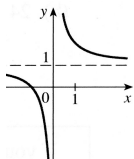
X.



Y.



Z.



- Izzy Sinkin winds up his toy boat and lets it run on the pond. Its speed is given by

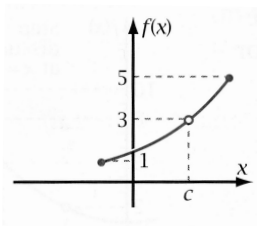
$$v(t) = (5t)(0.8)^t$$

where $v(t)$ is in feet per second and t is time in seconds since he let go.

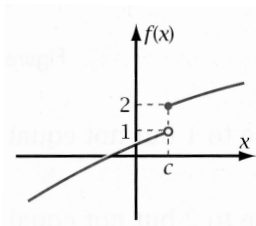
- Using your calculator to help, graph this function over the interval $t = [0, 10]$. Sketch the result.
- Find approximately the distance the boat travels between $t = 2$ and $t = 10$ using 4 equal width mid-point rectangles. Justify with clear and complete work.
- Repeat part (b) with 4 equal width trapezoids. Justify with clear and complete work.
- In terms of Calculus, what were you doing in parts (b) and (c) of this problem?

4. Tell whether or not the function has a limit as x approaches c ; if it does have a limit, tell what the limit equals using correct limit notation.¹

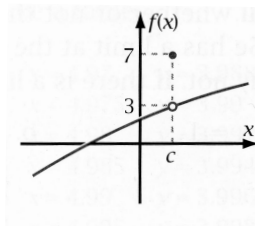
a.



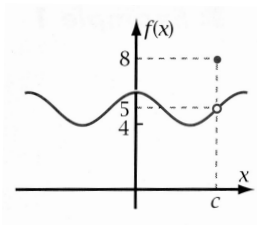
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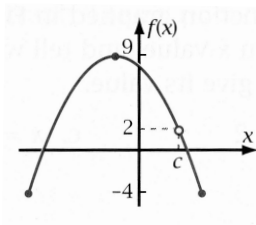
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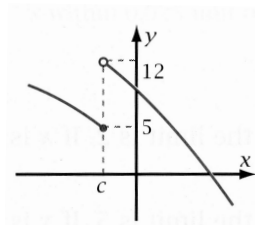
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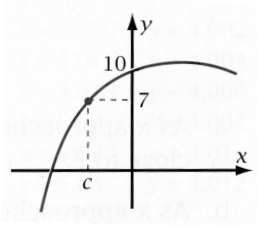
e.



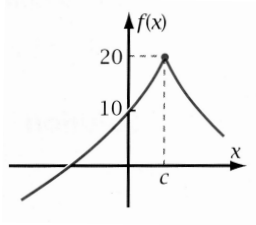
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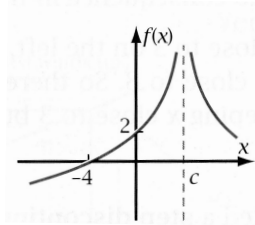
g.



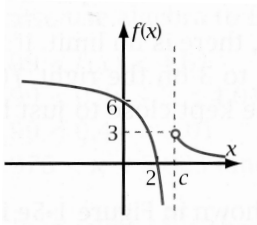
h.



i.



j.



5. Use your calculator to help evaluate the limits, to two decimal places, for each of the following. Express the limit using correct notation. If the limit does not exist, explain what happened.²

a. $\lim_{x \rightarrow 0^+} x^4$

b. $\lim_{x \rightarrow 0^+} \cos(x)$

c. $\lim_{x \rightarrow 2^-} (x^2 - 4)$

d. $\lim_{x \rightarrow 3^-} \left(\frac{1}{x-3} \right)$

e. $\lim_{x \rightarrow 3^+} \left(\frac{1}{x-3} \right)$

f. $\lim_{x \rightarrow 3} \left(\frac{1}{x-3} \right)$

g. $\lim_{x \rightarrow 3} \left(\frac{x^2 + 3x - 10}{x - 2} \right)$

h. $\lim_{x \rightarrow \pi} \left(\frac{\cos(x)}{x} \right)$

¹ From Foerster p.28

² From Bradely/Smith p. 96

6. Using algebra, show the following. Verify using your calculator.³

a. $\lim_{x \rightarrow 1} (x + 3) = 4$ b. $\lim_{x \rightarrow 1} \left(\frac{1}{x+2} \right) = \frac{1}{3}$ c. $\lim_{x \rightarrow 8} (x^2 - 7x + 5) = 13$

d. $\lim_{x \rightarrow 1} \frac{x^{\frac{3}{2}} - 2}{x^2 - 1} = \text{Does not exist}$ e. $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = 4$ f. $\lim_{x \rightarrow -2} \frac{(x+2)^2}{x^2 + 3x + 2} = 0$

g. $\lim_{x \rightarrow -1} \frac{x^2 - 1}{\sqrt[3]{x+1}} = 0$ h. $\lim_{x \rightarrow 1} \frac{1 - \frac{1}{x}}{x - 1} = 1$ i. $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \frac{1}{2\sqrt{x}}$

7. Sketch a graph to match each of the following descriptions and express the situation using limit notation.

- a. As x gets closer and closer to -4 from the left, $f(x)$ gets closer and closer to 3 .
- b. As x gets closer and closer to π from the right, $f(x)$ gets closer and closer to 1 .
As x gets closer and closer to π from the left, $f(x)$ gets closer and closer to 1 .
When x is equal to 1 , $f(x)$ is equal to -1 .
- c. $f(x)$ approaches -3 as x approaches 2 from the left.
 $f(x)$ approaches $+\infty$ as x approaches 2 from the right.
 $f(x)$ is equal to 0 when x is equal to 2 .

8. Evaluate the limit, if it exists. If you find an indeterminate form, use your creative algebra skills to manipulate it so that becomes “determinate”. If a limit does not exist, write “DNE” and state what went wrong. Justify with clear and complete work. Check with a graph.

a. $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2}$ b. $\lim_{x \rightarrow -1} \frac{x^2 - 4x}{x^2 - 3x - 4}$ c. $\lim_{h \rightarrow 0} \frac{(3+h)^3 - 27}{h}$

d. $\lim_{x \rightarrow -4} \frac{\frac{1}{4} + \frac{1}{x}}{4 + x}$ e. $\lim_{x \rightarrow 7} \frac{\sqrt{x+2} - 3}{x - 7}$ f. $\lim_{x \rightarrow 1} \left(\frac{1}{x} - 1 \right) \left(\frac{x}{x-1} \right)$

³ From Farrand/Poxon pp. 46-47