

# Intro to Calculus

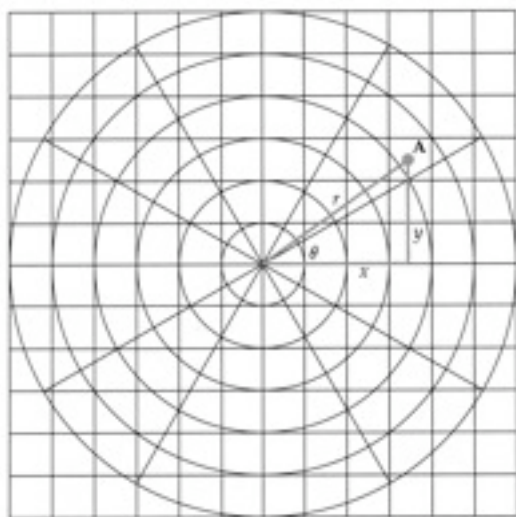
## Lesson for HW 26: To the Pole and Back!<sup>1</sup>

### Goals

I can convert between polar and rectangular coordinates.

Figure 1 shows rectangular and polar coordinate systems superimposed. The pole and the origin coincide, as do the polar axis and the positive  $x$ -axis. Notice that point  $A$ 's polar coordinates  $(r, \theta)$  and rectangular coordinates  $(x, y)$  describe the same right triangle.

Figure 1



1. Why is the  $x$ -coordinate of point  $A$  equal to  $r \cos(\theta)$ ?

2. What is the  $y$  coordinate of point  $A$  in terms of its polar coordinates?

3. Express the polar radius of point  $A$ ,  $r$ , in terms of point  $A$ 's rectangular coordinates.

4. Express the polar angle,  $\theta$ , in terms of point  $A$ 's rectangular coordinates.

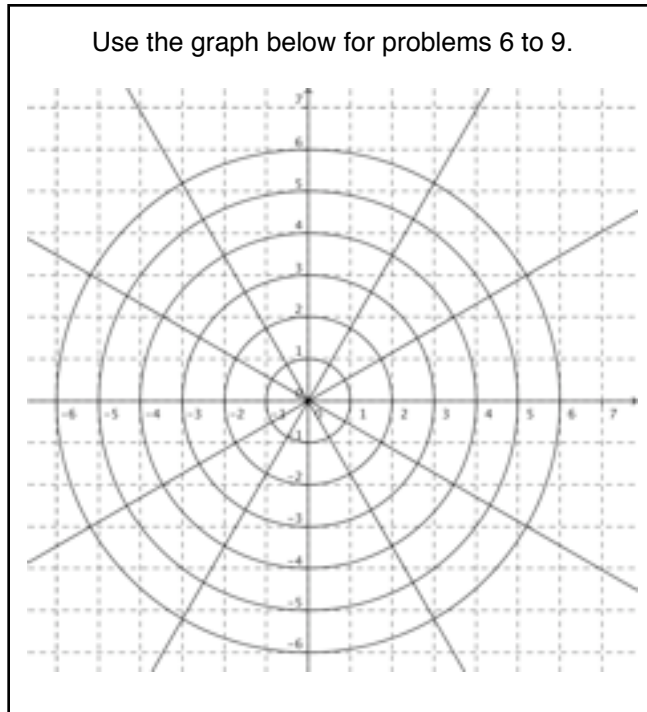
5. Convert the polar coordinates  $\left(2, \frac{\pi}{3}\right)$

to rectangular coordinates. Verify your results by plotting the points in the graph in Figure 1.

<sup>1</sup> Adapted from "Precalculus: Modeling Our World", COMAP Preliminary Edition, and "Pre-Calculus with Trigonometry", CPM

## Intro to Calculus: To the Pole and Back!

6. a. Plot point  $A$  with rectangular coordinates  $(2,3)$  to polar coordinates.  
b. Convert the rectangular coordinates to polar coordinates.
7. a. Plot point  $B$  with rectangular coordinates  $(-2,3)$  to polar coordinates.  
b. Convert the rectangular coordinates to polar coordinates.
8. a. Plot point  $C$  with polar coordinates  $\left(2, -\frac{\pi}{3}\right)$ .  
b. Convert the polar coordinates to rectangular coordinates.
9. a. Plot point  $D$  with polar coordinates  $\left(-4, -\frac{5\pi}{3}\right)$ .  
b. Convert the polar coordinates to rectangular coordinates.



## Intro to Calculus: To the Pole and Back!

10. Review your work from problems 6 to 9. Were there any surprises? What are you going to do when you are solving problems like these in the future?
11. a. Convert the rectangular equation  $x^2 + y^2 = 2x$  into a polar equation.
- b. Graph the polar equation on your calculator. Does it make sense?
11. a. Convert the rectangular equation  $y = x^2$  into a polar equation.
- b. Graph the polar equation on your calculator. Does it make sense?
12. a. Convert the polar equation  $r = 2 \sin(\theta) + \cos(\theta)$  into a rectangular equation.
- b. Graph the polar equation on your calculator. Does it make sense?

# Intro to Calculus

## Homework 26: To the Pole and Back!

### Problems

1. A point in polar coordinates is given. Find the corresponding rectangular coordinates without using a calculator. You might want to determine the quadrant of the point first.

a.  $\left(4, \frac{\pi}{2}\right)$       b.  $\left(2, -\frac{3\pi}{2}\right)$       c.  $\left(-1, \frac{5\pi}{6}\right)$       d.  $(0, -\pi)$

e.  $\left(-3, -\frac{\pi}{6}\right)$       f.  $\left(3, -\frac{11\pi}{6}\right)$       g.  $\left(-5, -\frac{28\pi}{3}\right)$       h.  $\left(5, \frac{28\pi}{3}\right)$

2. Find the rectangular coordinates of a point are given. Find two sets of polar coordinates for the point such that  $\theta \in [0, 2\pi)$ . Use your calculator to approximate non-special right triangle angles, otherwise state answers as exact values.

a. (1,1)      b. (-3,4)      c.  $(-\sqrt{3}, -\sqrt{3})$       d. (-3,0)      e. (0,-3)

3. Convert the following rectangular equations into polar equations in the form  $f(\theta) = r$ . (In other words, write the equation so that  $r$  is in terms of  $\theta$ .)

a.  $x^2 + y^2 = 4$       b.  $x^2 + y^2 = 9\left(\frac{y}{x}\right)^2$       c.  $3y - 7x = 10$       d.  $y = 2x - 3$

4. Convert the following polar equations into rectangular equations. The final form of your equation should be a standard form. Describe the shape of the graph giving specific details (like the center and radius of a circle).

a.  $r = 4 \sin(\theta)$       b.  $r = \sin(\theta) + \cos(\theta)$       c.  $r = -5 \sin(\theta)$

d.  $r = 4^2$       e.  $r = a^2 \mid a \in \mathbb{R}$

5. Points  $A$  and  $B$  are on circle  $O$ . In polar coordinates,  $A$  is located at  $\left(3, \frac{\pi}{4}\right)$  and  $B$  is located at  $\left(3, \frac{2\pi}{3}\right)$ . What is the length of chord  $\overline{AB}$ ?

6. Find the equation of a possible sinusoid that goes through a minimum of  $(-2, 3)$  and a maximum at  $(1, 5)$ .

Name \_\_\_\_\_

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**Selected Solutions**

3. a.  $r = \pm 2$    b.  $r = 3 \tan(\theta)$    c.  $r = \frac{10}{3 \sin(\theta) - 7 \cos(\theta)}$

4. a.  $x^2 + y^2 - 4y = 0$    c.  $x^2 + y^2 = a^2$