Name _____ Date _____ Class # _____ Block _____

# Intro to Calculus

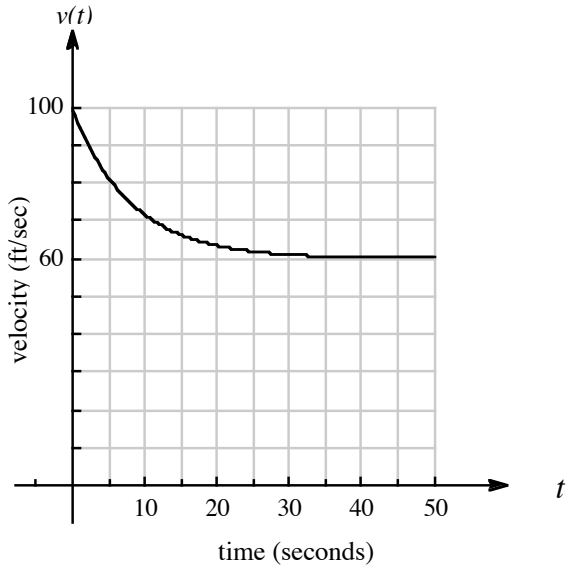
## Lesson for Homework 24

**Goals**

I can explain what is meant by the term "definite integral".

In the previous lessons you have learned about the derivative of a function and how this can be computed by finding the slope of the line tangent to the curve. You estimated this slope by using smaller and smaller Δx. In this lesson you will be introduced to the another main topic in calculus— the definite integral!

1. As you drive on the highway you accelerate to 100 feet per second to pass a truck. After you have passed, you slow down to a more moderate 60 $\frac{\text{ft}}{\text{sec}}$. The graph below shows your velocity, $v(t)$, as a function of the number of seconds, $t$, since you started driving.




   a. What does the velocity seem to be between $t = 30$ and $t = 50$ seconds?

   b. How far did you travel during the time interval [30, 50]?

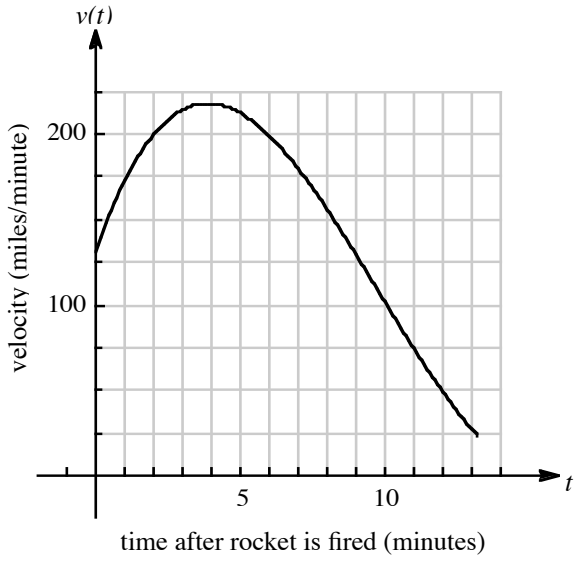
2. Explain why the answer to problem 1 can be represented as the area of the rectangular region under the curve and above the x-axis on the interval $t = [30,50]$. Shade this region.

3. a. The distance you travel between $t = 0$ and $t = 20$ can also be determined by finding the area under the curve and above the x-axis. Count the number of squares in this region to help find a rough estimate of the distance you travel.

   b. How many feet does each small square on the graph represent? How far, therefore, did you travel in the time interval $[0,20]$?

4. In the above problems finding the distance you traveled involved finding the product of the x-value and the y-value for a function where y may vary with respect to x. Such a product is called the definite integral of y with respect to x. Based on the units for time and velocity, explain why the definite integral of $v(t)$ with respect to $t$ has feet for its units.

5. Here is another situation. Suppose that Ella Vader (Darth's daughter) is driving in her rocket ship. At time $t = 0$ minutes she fires her rocket engine. The ship speeds up for a while, then slows down as Alderaan's gravity takes effect. The graph of her velocity, $v(t)$ miles per minute, is shown below.




   a. What mathematical concept would be used to estimate the distance Ella travels between $t = 0$ and $t = 12$?

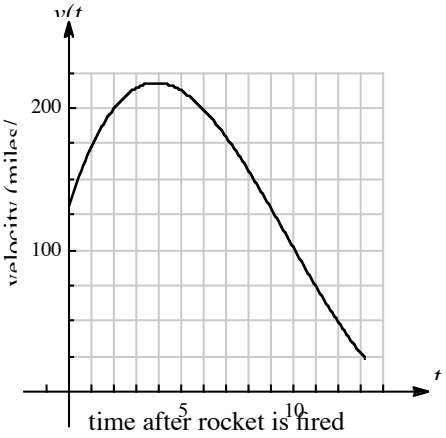
   b. Find a rough estimate for the distance traveled over this interval.

6. There are many ways to estimate the area under the curve and above the *x*-axis— many ways to estimate the definite integral. One way is to divide the region up into **rectangles**.

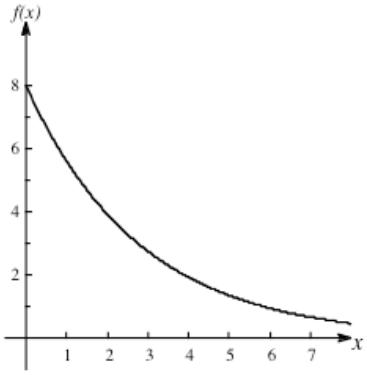
   Of course there are many ways to do this too! For now we will look at three different ways, all of which will use rectangles of equal width. Some students will have the **left corners** of their rectangles on the graph, some will have the **right corners**, and some will have the **midpoints** of the ends of the rectangles on the graph. All will use **four equal width** rectangles.

   Using the method you have been assigned, estimate the definite integral.

7. Now we can also use trapezoids to estimate the definite integral. Do this using four trapezoids of equal width.




8. Use trapezoids to estimate the definite integral of $f(x) = 8(0.7)^x$ from $x = 1$ to $x = 7$. Use 5 trapezoids. (Hint: The distance from $x = 1$ to $x = 7$ is 6 units. You want 5 strips of equal width, hence the width of each strip is…)




9. On a ship at sea, it is easier to measure how fast you are going than it is to measure how far you have gone. Suppose that you are the navigator aboard a supertanker. The speed of the ship is measured each 15 minutes and recorded in the table shown. Estimate the distance the ship has gone between 7:30 p.m. and 9:15 p.m. by first sketching a graph of the data and then using trapezoids to estimate the definite integral over the specified time interval.

| time | mi/hr | time | mi/hr |
|---|---|---|---|
| 7:30 | 28 | 8:30 | 7 |
| 7:45 | 25 | 8:45 | 10 |
| 8:00 | 20 | 9:00 | 21 |
| 8:15 | 22 | 9:15 | 26 |

10. Summary:

    a. What does “definite integral” mean?

    b. How can you estimate the definite integral given a graph of a situation?

    c. What operation are you performing on the variables when you find the integral?

    d. How can you determine the units of the definite integral?

    e. What operation are you preforming when you find the derivative of a function?

11. Here’s a quote for you:

    “The experimental verification of a theory concerning any natural phenomenon generally rests on the result of an integration.”-- J. W. Mellor

    As part of your homework find a formula or two from science that really is the result of an integration-- remember that you can tell if it’s an integration if you are multiplying the variables!