

Intro to Calculus

Lesson for Homework 22

Goals

I can estimate the instantaneous rate of change.

In this lesson we will analyze a situation concerning the motion a door. This situation will give us our first look at calculus and will preview key ideas that we will be working with all year.

1. Suppose that you push open a door that has an automatic closer. How does the motion of the door behave? Write a description and sketch a graph to represent the motion with respect to time.
2. For the sake of the lesson, let's assume that the door is pushed open at time $t=0$ sec and slams shut at time $t=7$ sec. While the door is in motion, assume that the number of degrees, d , from its closed position is modeled by the equation:

$$d = 200t \cdot 2^{-t} \quad \text{for } 0 \leq t \leq 7$$

Sketch a graph of this equation using your axes. Does the graph look reasonable?

3. How does the graph show that the door is **opening** at $t = 1$ sec?

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4. At what time is the door opening fastest? Slowest? How do you know?

5. How could I compute how fast the door is opening (the rate of change of the angle with respect to time) at $t=1$ sec? Or in other words, what is the instantaneous rate of change of the angle with respect to time at $t=1$ sec?

6. So how can we come up with a good approximation for the instantaneous rate of change of the angle with respect to time when $t=1$?

7. Find the average rate of change from $t=1$ to $t=1.1$?

8. Find the average rates over the following intervals:

Interval		average rate
x_1	x_2	(slope)
1	1.01	
1	1.001	
1	1.0001	
1	1.00001	
1	1.000001	

9. What's happening to the average velocity as we make the difference in x , Δx , smaller and smaller?

10. So what would you suggest that we do in order to get a really good estimate for the instantaneous velocity--the instantaneous rate of change of degrees with respect to time?

11. Now let's zoom in real close to $t=1$. What does the graph look more and more like as we get closer and closer?

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12. Now on your own come up with a good estimate of the instantaneous velocity of the door at $t=3$ sec. (Good meaning within $0.1 \frac{\text{radians}}{\text{second}}$ of the actual value.)

13. To summarize:

- a. For this situation, the angle is said to be a _____ of time.
- b. Time is the _____ variable and angle is the _____ variable.
- c. The instantaneous rate of change of the dependent variable is said to be the _____ of the average rates as the time interval gets closer and closer to zero.
- d. This limiting value is called the _____ of the dependent variable with respect to the independent variable.