

Intro to Calculus

Homework 22

Goals

I can estimate the instantaneous rate of change by using smaller and smaller intervals.

1. A ball is thrown down from the top of a building. The distance it has traveled after t seconds is $f(t) = 16t^2 + 20t$.
 - a. Find the average velocity of the ball over the interval $t = [2, 5]$.
 - b. Find the average velocity of the ball over the interval $t = [2, 4]$.
 - c. Find the average velocity of the ball over the interval $t = [2, 3]$.
 - d. Find the average velocity of the ball over the interval $t = [2, 2.5]$.
 - e. Find the average velocity of the ball over the interval $t = [2, 2.1]$.
 - f. Find the average velocity of the ball over the interval $t = [2, 2.01]$.
 - g. From your answers to the above, what do you expect will be the instantaneous velocity of the ball at time $t = 2$ seconds?
 - h. Find an expression for the average velocity of the ball between $t = 2$ and $t = 2 + h$
 - i. Simplify the expression you found in part h, and determine what happens to this simplified expression when h is very close to zero.

2. A pendulum hangs from the ceiling. As the pendulum swings, its distance, d , cm, from the wall of the room depends on the number of seconds, t , since it was set in motion. Assume that the equation for d as a function of t is

$$d = 80 + 30 \cos\left(\frac{\pi}{3}t\right), \quad t \geq 0$$

We want to find out how fast the pendulum is moving at a given instant, t , and whether it is approaching or going away from the wall.

- a. Find d when $t = 5$.
- b. Estimate the instantaneous rate of change of d at $t = 5$ by finding the average rates over the intervals $t = [5, 5.1]$, $t = [5, 5.01]$, and $t = [5, 5.001]$.
- c. Why can't the actual instantaneous rate of change of d with respect to t be calculated using the method used in problem 1b above?

Intro to Calculus: Homework 22

- d. Estimate the instantaneous rate of change of d with respect to t at $t = 1.5$. At that time is the pendulum approaching the wall or going away from it? Justify.
- e. How is the instantaneous rate of change related to average rates? What name is give to the instantaneous rate?
- f. What is the reason for the domain restriction $t \geq 0$? Can you think of any reason that there would be an *upper* bound to the domain?
3. Find all values of the variable which make the following equations true. Justify with clear and complete work.
- a. $2 \cos(2\theta) = 1$ b. $2 \tan^2(x) + \tan(x) - 3 = 0$ c. $\cos(2x) = 11\sin(x) + 6$
- d. $\sin(x) + \sqrt{3} \cos(x) = 1$