Name ________________ Date _______ Class # ______ Block ______

Intro to Calculus
Homework 21[1]

Goals
I can apply concepts and skills related to trigonometric functions to solve problems.

1. Suppose that $f(x)=\frac{1}{x}$. Suppose that the graph of $g(x)$ was the result of moving the graph of $f(x)$ right one unit, stretching it vertically by a factor of 2, and then moving the result up 5 units. What would be an equation for $g(x)$?

2. The graph of $g(x)=\frac{2}{x-2}+3$ is a transformation of the graph of $f(x)=\frac{1}{x}$.

   a. Describe the transformations, in terms of stretches/compressions and shifts.

   b. A student says that the graph of $h(x)=\frac{3x-4}{x-2}$ is the same as the graph of $g(x)$ from above. Can you show algebraically that this is true?

   c. How could the graph of $f(x)=\frac{1}{x}$ be transformed into the graph of $k(x)=\frac{2x+5}{x-2}$? Can you show algebraically that this is true?

3. Suppose that α is an angle in Q_I and β is an angle in Q_{IV} such that $\sin(\alpha)=\frac{2}{3}$ and $\cos(\beta)=\frac{1}{4}$.

   a. Find the exact value of $\cos(\alpha)$.
   b. Find the exact value of $\sin(\beta)$.
   c. Find the exact value of $\sin(\alpha+\beta)$.
   d. Find the exact value of $\cos(\alpha+\beta)$.
   e. Find the exact value of $\cos\left(\frac{\alpha}{2}\right)$.
   f. Verify your answers to the above using your calculator. No work needs to be shown for this.

[1] Based on material from Prof. Scott Farrand's Math 29B, CPM Pre-Calculus with Trigonometry pp. 449-470 with some additional problems from Algebra and Trigonometry Structure and Method pp. 699-700.

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4. If $\cos(x)=\frac{-4}{5}$, $x\in\left[\pi,\frac{3\pi}{2}\right]$, find the exact values of the five other trigonometric functions without using your calculator.

5. Simplify $\cos\left(\frac{\pi}{2}+\alpha\right)$ using the angle sum identity for cosine.

6. Solve the equation $4\cos^2(x)=3$ for $x\in(-\infty,\infty)$.

7. Simplify each of the following by applying the appropriate identity.

   a. $2\sin(3x)\cos(3x)$

   b. $\cos^2(40^\circ)+\sin^2(40^\circ)$

   c. $\cos^2(40^\circ)-\sin^2(40^\circ)$

   d. $1-2\sin^2(y-5)$

   e. $\sin(30^\circ)\cos(40^\circ)+\cos(30^\circ)\sin(40^\circ)$

   f. $2\cos^2(2w)-1$

8. Here is a neat way to use the double-angle formula! Suppose you were asked to solve the following equation:

$$\sin(x)\cos(x)=\frac{1}{4}$$

   Did you recognize that this would be difficult because the equation contains both sine and cosine? Use the double-angle formula for sine to help solve the equation! If you have trouble figuring out how, you can look for a hint in the "Hints and Sample Solutions" section of this assignment which is posted on-line.

9. If $\cos(x)=-\frac{4}{5}$, $x\in\left[\pi,\frac{3\pi}{2}\right]$, find exact values for $\sin(2x)$, $\cos(2x)$, $\sin\left(\frac{x}{2}\right)$, and $\cos\left(\frac{x}{2}\right)$. Be careful about getting the sign ($\pm$) right— a circle sketch might help.

10. Solve the equation $\sin(2x)=\cos(x)$, $x\in[0,2\pi]$ without using a calculator. Then verify your answer using a graphing calculator.

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11. Rewrite each expression in a simpler form using trigonometric identities.

    a. $2\sin(5p)\cos(5p)$

    b. $\sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right)-\cos\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right)$

12. Solve the following. Give exact answers where possible, otherwise approximate using your calculator to the nearest 0.01 unit.

    a. $\sin(x)(\cos(x)+1)=0, x\in[0,2\pi]$

    b. $x=\tan^{-1}(-\sqrt{3})$

    c. $\cos x(\tan(x)-1)=0$

    d. $\tan(x)+\sqrt{3}=0, x\in[0,2\pi]$

13. It is thought that $\frac{\cot(x)}{\sin(x)}(\sec(x)-\cos(x))$ always equals 1. Prove that this is true.

14. Graph $y=\frac{2x+7}{x-7}$ on your calculator.

    a. What is the domain of this function?

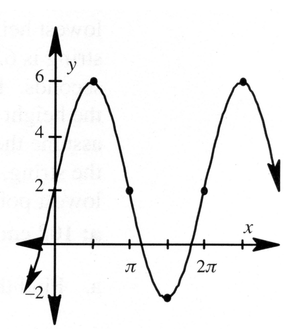
    b. What happens to *y* as you get really close to 7 while $x>7$? Why?

    c. What happens to *y* as you get really close to 7 while $x<7$? Why?

    d. What happens to *y* when *x* gets really, really, really big (and then some)? Why?

    e. What happens to *y* when *x* gets really, really, really small (and then some)? Why?

15. Use the graph shown for the following.

    a. Describe the transformations needed to change the graph of $y=\sin(x)$ into the graph shown.

    b. Write an equation, using the sine function, which would represent the graph.

    c. Repeat questions (a) and (b) using the cosine function.

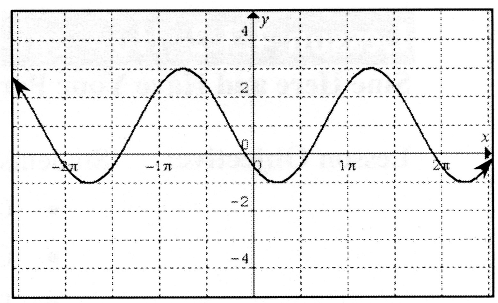
16. You know what the graph of $y = \sin(x)$ looks like. From this sketch the graphs of the following over the interval $0 \le x \le 2\pi$.

    a. $f(x) = 2\sin(x)$ b. $g(x) = \sin\left(x + \frac{\pi}{4}\right)$ c. $h(x) = \sin(2x) - 1$

    d. $k(x) = 3\sin(x - \pi)$

17. Using the smallest possible horizontal shift, find an equation for the graph to the right in terms of:

    a. $\sin(x)$ b. $\cos(x)$



18. Describe the transformations (in terms of stretches, compressions…) the given base graph would go through to become the graph of each of the following.

    a. $g(x) = 5\sin(3x)$; base function $f(x) = \sin(x)$

    b. $h(x) = -3\cos(\pi x) + 2$; base function $f(x) = \cos(x)$

    c. $k(x) = \cos\left(\frac{\pi x}{5}\right) + 3$; base function $f(x) = \sin(x)$

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Hints and Sample Solutions

2b. Simplify $\frac{2}{x-2}+3$ and the result should be $\frac{3x-4}{x-2}$.

2c. Examine what happened in parts a and b of this problem. Try to see how the values in the simplified version of the equation, $\frac{3x-4}{x-2}$, are related to the original equation $\frac{2}{x-2}+3$. In other words, try and see how where the $3x$ came from and where the -4 came from. Then use this knowledge to work backwards from $\frac{2x+5}{x-2}$. It might take you some trying and revising!

8. The double angle formula for sine is

$$\sin(2t) = 2\sin(t)\cos(t)$$

and the equation you are given to solve is

$$\sin(x)\cos(x) = \frac{1}{4}.$$

So you can multiply each side of the equation by 2 to give you

$$2\sin(x)\cos(x) = 2\left(\frac{1}{4}\right)$$

$$2\sin(x)\cos(x) = \frac{1}{2}$$

And by the double angle identity you get

$$\sin(2x) = \frac{1}{2}$$

By substitution $\alpha = 2x$ this equation becomes

$$\sin(\alpha) = \frac{1}{2}$$

Which yields $\alpha = \frac{\pi}{6}, \frac{5\pi}{6}$ and hence $x = \frac{\pi}{12}, \frac{5\pi}{12}$.