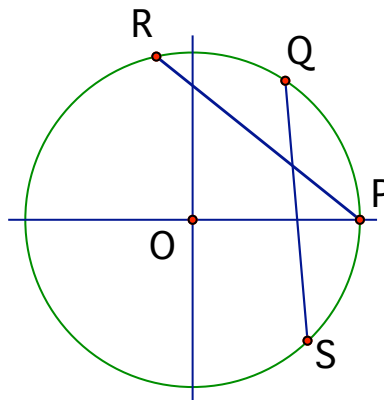


# Intro to Calculus

## Lesson for HW 20

### Cosine of a Sum<sup>1</sup>

Let  $O$  be located at  $(0,0)$  and  $P$  be the point with coordinates  $(1,0)$ . Let  $Q$  be the terminal point of angle  $\alpha$ , let  $R$  be the terminal point of angle  $\alpha + \beta$ , and let  $S$  be the terminal point of angle  $-\beta$ .



1. Draw, and label, angles  $\alpha$ ,  $\beta$ ,  $\alpha + \beta$ , and  $-\beta$  on the diagram.
2. What are the coordinates of points  $Q$ ,  $R$ , and  $S$  in terms of  $\alpha$  and  $-\beta$ ?
3. What's true about the lengths of segments  $PR$  and  $QS$ ? Why?
4. What's the distance between points  $P$  and  $R$ ? Between  $Q$  and  $S$ ?

5. Use your answers from above to clearly and completely show that  $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$ . (You might want to just start this on another sheet of paper — even turn the paper sideways since it really stretches out.)

### Goals

I can explain how to rewrite the cosine of the sum of two angles in terms of the angles themselves and I can explain why this works.

### Vocabulary

I am tired of writing long sentences describing the "point of intersection of the terminal side of the angle and the (unit) circle whose initial side is located on the positive x-axis". Let's just call this the **terminal point** and be careful about declaring if we are working with the unit circle.

<sup>1</sup> Based on work from Prof. S. Wilson <http://staff.jccc.edu/swilson/trig/anglesumidentities.htm>

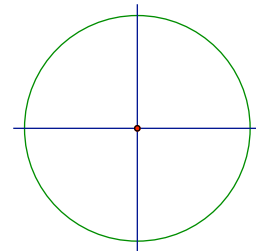
## Intro to Calculus: Lesson for Homework 20

### Sine of a Sum

6. For each problem below, show and clearly label both angles.

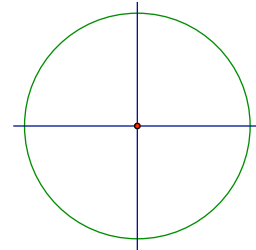
a. Let  $A\left(\frac{5}{13}, \frac{12}{13}\right)$  represent the terminal point of angle  $\alpha$ .

Find the coordinates of the terminal point of  $\frac{\pi}{2} - \alpha$ .



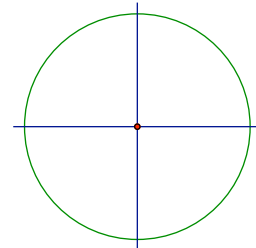
b. Let  $B\left(\frac{-5}{13}, \frac{12}{13}\right)$  represent the terminal point of angle  $\beta$ .

Find the coordinates of the terminal point of  $\frac{\pi}{2} - \beta$ .



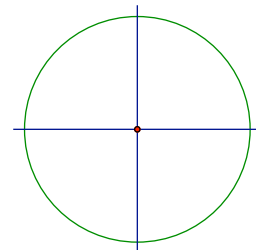
c. Let  $D\left(\frac{-5}{13}, \frac{-12}{13}\right)$  represent the terminal point of angle

$\delta$ . Find the coordinates of the terminal point of  $\frac{\pi}{2} - \delta$ .



d. Let  $E\left(\frac{5}{13}, \frac{-12}{13}\right)$  represent the terminal point of angle

$\varepsilon$ . Find the coordinates of the terminal point of  $\frac{\pi}{2} - \varepsilon$ .



7. Review your data from problem 6. What patterns do you notice?

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8. Let  $\sin(\alpha) = \cos(\beta)$ . Express  $\beta$  in terms of  $\alpha$ .
9. Use a drawing of a right triangle to help explain why your answer to problem 8 is correct (words  $\geq 20$ ).
10. Using your work from above, complete the following:  
 $\sin(\alpha + \beta) = \cos(\text{_____})$
11. Using the fact that  $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$ , the ideas from the previous problems, and your prior knowledge, show that  $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$ . In other words, starting either side (left is easier) clearly show how to turn it into the other side.