Name _____ Date _____ Class # _____ Block _____

Intro to Calculus

Homework 20[1]

Goals

I can work with trigonometric expressions which include arguments involving sums and differences.

1. Write a sentence describing what each of the following is asking, then evaluate without using your calculator. Restrict angles to be in $(0,2\pi]$. Note: A sketch of the unit circle might help you make sure you have all of the solutions.

   a. $\sin^{-1}\left(\cos\left(\frac{\pi}{8}\right)\right)$

   b. $\cos^{-1}\left(\sin\left(\frac{4\pi}{7}\right)\right)$

   c. $\sin^{-1}\left(\cos\left(\frac{-11\pi}{9}\right)\right)$

   d. $\cos\left(\sin^{-1}\left(\frac{4}{5}\right)\right)$

   e. $\sin\left(\cos^{-1}(-0.2)\right)$

2. A rocket shoots straight up from the launchpad. Six seconds after liftoff an observer 3 miles away notes that the rocket's angle of elevation is 4.2˚. Five seconds later the angle of elevation is 45˚. How far, to the nearest 0.001 miles, did the rocket rise during those 5 seconds?

3. Using the fact that $\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$ show that $\cos(\alpha-\beta)=\cos(\alpha)\cos(\beta)+\sin(\alpha)\sin(\beta)$.

4. Using the fact that $\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$, show that $\cos(2t)=\cos^2(t)-\sin^2(t)$.

5. Using the fact that $\sin(\alpha+\beta)=\sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta)$, show that $\sin(2t)=2\sin(t)\cos(t)$.

6. Show that $\cos^2(t)=\frac{1+\cos(2t)}{2}$. (Hint: Start with the double angle formula for cosine— which you proved in problem 4— and use the Pythagorean Identity to replace the $\sin^2(t)$ term and then solve for $\cos^2(t)$.)

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[1] Based on Prof. Scott Farrand's Math 29B Homework 9 and on work from G.Rising Unified Mathematics Book 3, pp.436- 437

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7. Show that $\sin^2(t) = \frac{1-\cos(2t)}{2}$. (Hint: Start by using the Pythagorean Identity to re-write $\sin^2(t)$).

8. Find an exact value for $\cos\left(\frac{\pi}{12}\right)$ by using the fact that $\frac{\pi}{12} = \frac{\pi}{4} - \frac{\pi}{6}$.

9. Find the exact value of $\sin\left(\frac{5\pi}{12}\right)$ by using sum or difference formulas.

10. Find the exact value of following by first writing it in terms of the sine or cosine of the sum of two angles.

    a. $\sin\left(\frac{11\pi}{18}\right)\cos\left(\frac{\pi}{9}\right) - \cos\left(\frac{11\pi}{18}\right)\sin\left(\frac{\pi}{9}\right)$

    b. $\cos\left(\frac{2\pi}{9}\right)\cos\left(\frac{\pi}{18}\right) + \sin\left(\frac{2\pi}{9}\right)\sin\left(\frac{\pi}{18}\right)$

    c. $\sin\left(\frac{\pi}{9}\right)\cos\left(\frac{5\pi}{36}\right) + \cos\left(\frac{\pi}{9}\right)\sin\left(\frac{5\pi}{36}\right)$

11. Use the given information (and the identity for the sine of a sum) to find the $\sin(\alpha+\beta)$ if α and β are in Q_I.

    a. $\sin(\alpha) = \frac{3}{5}$, $\sin(\beta) = \frac{5}{13}$

    b. $\cos(\alpha) = \frac{7}{25}$, $\cos(\beta) = \frac{4}{5}$

    c. $\sin(\alpha) = \frac{56}{65}$, $\tan(\beta) = \frac{63}{16}$

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12. Prove each of the following identities.

    a. $\cos(\alpha-\beta)+\cos(\alpha+\beta)=2\cos(\alpha)\cos(\beta)$

    b. $\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin(\alpha)\cos(\beta)$

    c. $\dfrac{2\cos(\theta)}{1+\cos(2\theta)}=\sec(\theta)$

13. Use the given information to find the exact value of $\cos(2\theta)$.

    a. $\cos(\theta)=\dfrac{1}{3}$ b. $\sin(\theta)=-\dfrac{4}{5}$ c. $\sin(\theta)=\dfrac{\sqrt{5}}{3}$ d. $\tan(\theta)=\dfrac{5}{12}$

14. Use the given information to find the exact value of $\sin(2\alpha)$.

    a. $\sin(\alpha)=\dfrac{5}{13},\ 0<\alpha<\dfrac{\pi}{2}$ b. $\cos(\alpha)=\dfrac{8}{17},\ 0<\alpha<\dfrac{\pi}{2}$

    c. $\sin(\alpha)=\dfrac{\sqrt{7}}{4},\ \dfrac{\pi}{2}<\alpha<\pi$ d. $\sin(\alpha)=-\dfrac{\sqrt{15}}{4},\ \pi<\alpha<\dfrac{3\pi}{2}$

15. Find all values that make each of the following true. Justify with clear and complete work. (Be careful not to loose solutions!)

    a. $\sin(\alpha)=\sin(2\alpha)$ b. $\cos(2\alpha)=\cos(\alpha)$

16. What are the identities that you used for doing the problems on this assignment? Can you recall them without looking? Do you have a good way to remember them?

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Selected Solutions

1. First a note-- later on in the course we will talk further about the domain of the inverse trigonometric functions (you may recall this from previous courses). For the moment I decided to avoid these eventual complications and just let the domain for these angles be $(0,2\pi]$. With that said:

   a. $\sin^{-1}\left(\cos\left(\frac{\pi}{8}\right)\right)$ asks the question, "What's the angle whose sine equals the cosine of the angle $\frac{\pi}{8}$?" In the specified domain these angles are $\frac{\pi}{8}+\frac{\pi}{2}=\frac{5\pi}{8}$ and $\frac{\pi}{8}-\frac{\pi}{2}=\frac{-3\pi}{8}$.