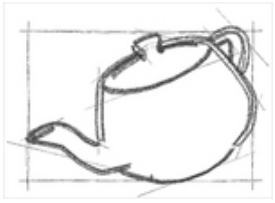
Name _____ Date _____ Class # _____ Block _____

Intro to Calculus
Working with Logs

Goals

I can explain the meaning of a logarithm.

I can solve problems which involve logarithms by translating them to exponent problems.



A mathematician was in a habit of making a cup of tea when working late at night. His normal method was to get the teapot from the cupboard, take the teapot to the sink, add water, heat to boiling, then make the cup of tea. Unfortunately, one night when he went to make tea, the teapot was already full of water and sitting on the stove. He thought about this for several minutes, then emptied the teapot and put it back in the cupboard, thereby reducing this to a previously solved problem.

—Origin unknown

Noodle Review

Let's return to our Beijing noodle maker. First a quick review.

1. Suppose that our noodle maker doubled his noodles with each stretch and started with one noodle. How many noodles would he have after 10 stretches?

Now suppose that the noodle maker was not so interested in knowing how many noodles he would have after 10 stretches. What he really wants to know is how many stretches it is going to take him to until he can drop them in the pot to boil.

2. Suppose that it is a big pot and that it holds about 1,200 noodles. About how many stretches will he have to make before he can drop the noodles?

So we now have two types of questions:

(I) If you know the number of stretches, can you predict the number of noodles.

(II) If you know the number of noodles you want, can you predict how many stretches it will take to make them.

Considering our two situations above, we can abstract the key relationships to write the equations:

$2^{10} = x$ where x represents the number of noodles

$2^x = 1200$ where x represents the number of stretches

As a diagram we can see that in the first case we are given stretches and want noodles, and in the second we are given noodles and want stretches. We can represent these relationships as

$$\textit{stretches} \rightarrow \textit{noodles}$$
$$\textit{noodles} \rightarrow \textit{stretches}$$

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3. How are the two types of noodle problems related to each other?

Representing the Situations
Here are three ways to represent the second noodle situation:

(I) I want to know the number of stretches it will take to make 1200 noodles.

(II) $2^x = 1200$

(III) $\log_2 1200 = x$

4. In your own words, what does $\log_2 1200 = x$ mean?

So an equation involving logarithms is actually an exponential equation in disguise. More specifically, **an equation involving logarithms is an exponential equation where the exponent is an unknown value.** This gives us a method by which we can solve problems involving logarithms— change the logarithm problem into an exponent problem and then solve that problem. Let's try this.

Rewrite each of the following as an equation involving exponents, then solve it. **No calculators.**

5. $\log_2 32$

6. $\log_3 81$

7. $\log\left(0.01\right)$

8. $\log_9\left(27\right)$

9. $\log_{\frac{1}{5}}\left(\sqrt[3]{5}\right)$

10. $\ln\left(\frac{1}{\sqrt[5]{e}}\right)$

11. $b^{\log_{\sqrt{5}}(5)}$

12. $b^{\log_{\sqrt[3]{b}}(3)}$

13. $\ln\left(e^{\frac{2}{3}}\right)$

Wrap Up

14. What are the two types of exponential situations we are dealing with (think about the noodle questions)?

15. How do logarithms connect to noodle making?

16. What is one way to deal with an equation involving logarithms?