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Intro to Calculus
Homework 13[1]

Goals
I can work with expressions and equations involving negative exponents.

1. Simplify. Leave no negative exponents in your final solution. Justify with clear and complete work.
   a. $(x-8)^4\left[-3-(x-8)\right]$
   b. $x^{-4}(11x-1)$
   c. $x^{\frac{-12}{9}}(-9x+1)$
   d. $(x-4)^{-9}\left[-5(x-4)+1\right]$
   e. $(7-x^4)^{\frac{-8}{5}}\left[(7-x^4)+3x^4\right]$
   f. $(3x-5)^{\frac{-29}{6}}\left[7(3x-5)-4x\right]$
   g. $(x-10)^{\frac{-7}{4}}\left[(3x+4)(x-10)-(5-x)\right]$

2. Determine if the function is odd, even, or neither. Justify with clear and complete work.
   a) $f(x)=7^{x^2+4}$
   b) $f(x)=x4^{-x^2}$
   c) $f(x)=\dfrac{e^x+e^{-x}}{2}$
   d) $f(x)=\dfrac{e^x-e^{-x}}{2}$
   e) $f(x)=8^{x^3}$
   f) $f(x)=x8^{x^3}$

3. Determine the domain of the function. Justify with clear and complete work.
   a) $f(x)=\dfrac{1}{5^{x+1}}$
   b) $f(x)=\dfrac{1}{3^x-1}$
   c) $f(x)=\dfrac{31}{e^{x+2}-1}$
   d) $f(x)=\dfrac{-14}{1-746^{2x-24}}$
   e) $f(x)=\dfrac{31}{e^{x+2}-e^2}$
   f) $f(x)=\dfrac{4}{7^{3x^2+x}-49}$

4. Perform the compositions. Justify with clear and complete work.

$$f(x)=\frac{3+x}{7},\qquad g(x)=e^{x+2},\qquad h(x)=\sqrt{\frac{x}{x+4}}$$

   a) $f\big(g(x)\big)$
   b) $g\big(f(x)\big)$
   c) $g\big(h(x)\big)$
   d) $h\big(g(x)\big)$

[1] Great thanks to Prof. Scott Farrand's (CSUS) Math 29B Materials and to David Danielson (Butte College)

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5. The number of bacteria growing in a contaminated piece of meat doubles each day while in the refrigerator. When you buy this meat, it contains 300 bacteria.
   a) Find a function of the form $f(t) = Cb^t$ that represents the number of bacteria in the meat after *t* days. (Assume the meat was forgotten in the back of the refrigerator.)
   b) How many bacteria are there after 2 weeks?
   c) Use a graph to find, approximately, how many days will it take for the number of bacteria to reach 1 million?

6. The half-life of carbon-14 is 5730 years. You initially have 7 grams of carbon-14.
   a) Show how to use the initial amount of carbon-14 and the half-life to create a rule to determine the amount of carbon-14 left after *t* years.
   b) How much carbon is left after 100 years? After 1000 years? After 1,000,000 years?

7. If the inflation rate remains at 4% per year, then the amount a dollar is worth decreases by 4% per year.
   a) Let's assume that today a dollar is actually worth $1.00. At the stated rate of inflation, how much will this dollar be worth after 6 years?
   b) Find a function which describes the value of a dollar after *t* years.

8. A substance loses 7% of its mass every hour. Let *m* represent the initial mass of the substance. In terms of *m*, how much of its mass remains after *t* hours?

9. A substance loses 7% of its mass every 3 hours. How much of its initial mass, *m*, remains after *t* hours?

10. The number of cells in a lab experiment triples every 15 minutes. The experiment began with 2 cells.
   a) Find a function that describes the number of cells present after *t* minutes.
   b) Find a function that describes the number of cells present after *t* hours.
   c) Using your two functions, show that you arrive at the same answer for $t = 2$ days.

11. A substance initially weighs 12 grams, but is exponentially decaying. After 1 day, the substance weighs 10 grams.
   a) Make a table of values for days 0 to 5.
   b) Find a function that describes the amount of the substance remaining after *t* days.

12. A yeast colony is growing exponentially. It starts with 400 individuals and 2 hours later it has 4,000 individuals. Find a function that describes the amount of yeast in the colony after *t* hours.

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Sample solutions:

2a. $f(-x) = 7^{(-x)^2+4} = 7^{x^2+4} = f(x)$ so f is even

3c. denominator is zero when $e^{x+2} = 1$, and thus $x + 2 \neq 0,\ x \neq -2$

4c. $g\left(h(x)\right) = g\left(\sqrt{\frac{x}{x+4}}\right) = e^{\sqrt{\frac{x}{x+4}}+2} = e^{2+\sqrt{\frac{x}{x+4}}}$

5a. $f(0) = 300$ and $f(t) = Cb^t$ so $300 = f(0) = Cb^0 = C$

Thus $f(t) = 300b^t$. Because it doubles every day,

$f(1) = 2 \bullet 300 = 600$ and so $600 = f(1) = 300b^1$. Solving for b we find b=2.

$f(t) = 300 \bullet 2^t$

b. $f(7) = 300 \bullet 2^7 = 38{,}400$ bacteria

9. $f(t) = Cb^t$ We are told that f(1)=.93f(0), so $Cb^3 = .93 \bullet Cb^0$. Solving for b we find $b^3 = .93$ so $b = \sqrt[3]{.93} \approx .976$. After t hours, the amount that remains is $f(t) = C(.976)^t$.